

XIX. *An Account of the Mode proposed to be followed in determining the relative Situation of the Royal Observatories of Greenwich and Paris. By Major-General William Roy, F. R. S. and A. S.*

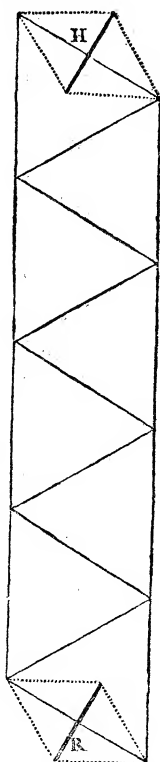
Read February 22, 1787.

TWO years have nearly elapsed since an account of the measurement of a base on Hounslow-Heath was laid before the Royal Society, being the first part of an operation ordered by his Majesty to be executed for the immediate purpose of ascertaining the relative situations of the Royal Observatories of Greenwich and Paris; but whose chief and ultimate object has always been considered of a still more important nature, namely, the laying the foundation of a general survey of the British Islands.

When the operation commenced in 1784, it was not doubted, that in 1786, at latest, we should have been able to have proceeded with the series of triangles from Hounslow-Heath to the neighbourhood of Dover; but the contrivance and construction of an instrument, new of its kind, proposed to be made use of, and more particularly the nicety of its division, whereby it is hoped the angles may be determined to a degree of precision hitherto unexampled, have required much more time than Mr.

RAMSDEN himself at first imagined. Without meaning to disappoint, this ingenious artist was perhaps in the outset too remiss and dilatory, and accidents having happened when the workmanship was already far advanced, which he could not foresee or prevent, the execution has thereby been greatly retarded. However, since the instrument may at present be considered as nearly finished (such parts as yet remain to be perfected being only of the smaller kind), we may fairly conclude, that early in the ensuing summer, or as soon as the weather in this country will permit, the trigonometrical operation may be begun. In this state of things, I have therefore judged that it might be proper to lay before the Society a short sketch of the mode which is proposed to be followed in fulfilling his Majesty's commands, accompanied by a very slight general map of the country, only collected from the common surveys, but still sufficient to shew nearly the disposition of the triangles that will be made use of in forming the junctions between the meridians of the two Observatories. In this business it will be understood, that I mean to adhere to such principles as have been universally received and admitted as just.

In every series of triangles where each angle is to be actually observed with the same instrument, they should, as near as the circumstances will permit, be equilateral: for were it possible to choose the stations in such a manner as that each angle should be exactly 60 degrees, the half number of triangles in the series, multiplied by the length of one side, would, as in the annexed figure, give at once the total distance; not only the sides of
the



the scale or ladder would be perfectly parallel, but the diagonal steps, marking the progress from one extremity to the other, would be alternately so throughout the whole length. The first side is supposed to be found by the measurement of a base *H*, of about half its length; and the last side to be verified by such another base *R* at the opposite extremity.

In any particular case, where only two angles of a triangle can be actually observed, these should be as near as possible each 45 degrees. At any rate their sum should not differ much from 90°; for the less the computed angle differs from 90°, the less chance there will be of any considerable error in the intersection.

Romney-Marsh, from its levelness, as well as other advantageous circumstances attending its situation, which the bare inspection of the map will render obvious, seeming to me to afford the best base of verification for the last triangle, I have given the series the shortest direction from Hounslow-Heath to that part of Kent. The right hand stations occupy in general the heights which extend across the Wealds. Those on the left are placed on the great range of chalk hills, which end, on our side of the Channel, between Folkestone and Walmer-Castle, and re-commence on the opposite side between Cape Blancnez and Calais.

It will be perceived, that I do not mean to make St. Paul's a station in the suite; because in that case Harrow and Hampstead must likewise have been made use of, all three extremely inconvenient for the reception of the great instrument. Besides, Greenwich Observatory being hidden from the country to the south-west by the Norwood heights, and from that to the south-east by Shooter's-Hill, after having made the detour of Harrow and Hampstead, and come across the smoke of the
Capital,

Capital, we should still have been obliged to make use of the two stations of Norwood and Shooter's-hill, without procuring so good an interfection of Bottle-hill, called in the common maps (I believe erroneously) Botley-hill, as is obtained by means of the station at the Hundred-acre-house. But although none of the stations of the series actually fall within London, nevertheless, from those in its vicinity, *viz.* the Pagoda, Norwood, Greenwich Observatory, and Shooter's-hill, we shall equally have it in our power to determine accurately the situations of Harrow, Hampstead, and St. Paul's, as well as many other chief steeples within the limits of the Capital.

Another principle I have endeavoured to adhere to, in the disposition of the triangles, is this, that, after having obtained sides in length from 12 to 18 miles, I continue them at that length as much as the circumstances will permit; for, if they came to be reduced considerably below that extent, the obvious advantages of a long base at the outset would be lost, by the subsequent contraction towards the close of the operation.

The tower of Tenterden church, being a very conspicuous object, may be seen every where from the summit of the chalk hills, as far west as the river Medway. It likewise may be seen from the eastern extremity of the second base, whereby the last triangle, *Tenterden, Lid, Allington Knoll*, is proposed to be verified. This knoll is itself a very remarkable object, more accessible, and in other respects more proper too for the purpose of a station than Lymne church steeple, which I had at one time thoughts of occupying. The high ground which separates Romney-marsh from the Wealds of Kent, passes immediately behind *Ruckinge*, that is to say, to the north-westward of it, and may therefore probably prevent the top of the chalk hills from being seen from the west end of the base

of verification; but if *Tatterlees-Barn**, or any other point on the range near it, can be seen from Ruckinge, then the station on the knoll, as well as that at Lymne, will become equally unnecessary, and the triangle of verification will become *Tenterden, Lid, Tatterlees*.

It will be perceived, that I propose to have a station on *Fairlight Head*, a land of considerable height, from whence there is a good view of the coast of France near Boulogne. From this point and Tatterlees, with the help of the Indian lights, I have no doubt of obtaining a fine intersection of the signal of the *Bouleberg*, a hill of some note behind the town of Boulogne, and one of the stations made use of by the French Academicians in the execution of their triangles. The advantages of obtaining a triangle of this magnitude, whose sides are respectively in length about 45, 36, and 25 miles, are too obvious to require any comment.

The high chalk cliffs near Folkestone prevent *Dover Castle* from being seen from Lid, or any where in the plain of Romney-marsh. Hence it will become necessary to form two small triangles to the northward of Tatterlees, in order to obtain an intersection of one of the turrets of the keep of that castle. Of the center of the keep, it will be perceived, by the strong dotted lines, that the French Academicians have procured, from their stations at *Calais, Blancnez, and Audinghen*, an acute intersection (*de la Grosse Tour de Douvres*) making in the whole an angle of $28^{\circ} 16' 20''$.

The points which are obviously the best for connecting our triangles with those of our neighbours, are the *Bouleberg, Blancnez, and Calais*, provided we could by any means obtain

* Tatterlees Barn, in *PACKE'S Map of East Kent*, on the summit of the chalk hills, 727 feet above the sea.

as good an intersection of the last as we are certain of getting of the two first; but the breadth of the range of chalk hills being little different on our side from what it is on theirs, by confining ourselves to such a base as they will afford us, we cannot any way obtain an intersected angle at Calais greater than about 29° or 30° .

Thinking that possibly from St. Peter's church in the isle of Thanet the tower of *Notre Dame* at Calais may be seen, I have extended dotted triangles into that part of Kent; because, if the united heights should not be sufficient to raise the top of the tower above the curvature of the sea, which is the only thing to be doubted, we are always certain, that the signal of *Blancnez* may, by means of the Indian lights, be easily seen, since the whole range of chalk hills behind Calais are discovered with the naked eye from the isle of Thanet, when the weather is tolerably clear.

Having in this manner ascertained the relative situations, with regard to the coast of England, of three points on the coast of France, forming a triangle whose sides and angles we already know from their trigonometrical operations, we shall in like manner be enabled to determine the situation of the point M near Dunkirk, where the meridian of the Royal Observatory of Paris intersects a line drawn from the great tower of Dunkirk to that of *N. D.* at Calais. (See Tab. IX.) The distance MP, on the meridian of Paris, will then be had; and that being added to 133417 fathoms, the distance of M northward from the Royal Observatory, we shall have the total terrestrial arc, comprehended between the parallels of the two Observatories, answering to an arc in the heavens of $2^{\circ} 38' 26''$, or a difference of latitude between $51^{\circ} 28' 40''$ and $48^{\circ} 50' 14''$.

In like manner the distance from Greenwich to P, on the parallel of Greenwich, will then be readily computed, answering to the difference of longitude between the two Observatories; which, as far as can be judged from the map of Kent, corrected for the error in the direction of its meridian, amounts to about $2^{\circ} 20' 20''$, supposing always that no uncertainty remains with regard to the position of the point M. But here some remarks become necessary, which may probably suggest to the Academy of Sciences, that a further investigation of this matter may be needful on their part.

By referring to the 57th page of the first part of M. CASSINI's Book (*La Méridienne vérifiée*), it will be seen, that Dunkirk, by one series of triangles, is eastward from the meridian of Paris 1426.53, and by another 1414.29 toises, whereof the mean is 1420.41, equal to 1514 fathoms. This difference of $6\frac{1}{2}$ fathoms, or little more than half a second of longitude between the mean and extreme places of M, is certainly very inconsiderable. But in the 60th page, where, in verifying the meridian of Paris, by the comparison of the angle that *Broulezele* makes with the meridian of Dunkirk, and the angle of convergence of one meridian to the other, a difference of 21 seconds between $10^{\circ} 16' 13''$ and $10^{\circ} 16' 34''$, is alledged to be almost insensible, we do not think to be a conclusion so unexceptionable. This, however, is not the only cause of uncertainty with regard to the just position of the point M: one of more importance arises, from the difference that is found by two sets of triangles in the angle of intersection of the meridian of Paris, with a line drawn through M from the tower of Dunkirk to that of Calais.

Thus, by p. 53. and 56. of the first part of M. CASSINI's book, Dunkirk being the station, *Broulezele* makes an angle with

with the meridian of $10^{\circ} 18' 25''$ towards the south-west: and the angle between Broulezele and Hondscote being $78^{\circ} 11' 42''$, their difference $67^{\circ} 53' 17''$ is the angle that Hondscote is south-east from the meridian; wherefore the complement of this last angle to 180° , *viz.* $112^{\circ} 6' 43''$ is the angle that Hondscote makes with the meridian of Dunkirk produced northward. By p. 166. of the second part, Dunkirk being the station, the angle between Hondscote and Mont-Cassel is shewn to be $51^{\circ} 7' 15''$; that between Mont-Cassel and Watten $42^{\circ} 6' 35''$; and by p. 167. that between Watten and Calais is $51^{\circ} 40' 20''$. The sum of these three angles is $144^{\circ} 54' 10''$, from which deducting $67^{\circ} 53' 17''$ the angle that Hondscote is south-eastward from the meridian, there remain $77^{\circ} 0' 53''$ for the angle of Calais south-westward from it; and the complement of this angle to 180° , *viz.* $102^{\circ} 59' 7''$, becomes the angle that the meridian of Dunkirk produced northward makes with a line drawn through M to Calais: to which last adding the angle of convergence of one meridian to the other $1' 50''\frac{1}{2}$, corresponding to the distance of 1514 fathoms, equal to $1' 29''\frac{1}{2}$ of a great circle, we shall have $103^{\circ} 0' 57''\frac{1}{2}$ for the angle which the meridian of Paris produced northward from M makes with the line joining Dunkirk and Calais.

Again, by p. 63. of the third part of M. CASSINI's book, Dunkirk being the station, the angle that Gravelines makes with the meridian south-westward is $72^{\circ} 11' 48''$; and by p. 12. of the said third part, the angle between Watten and N. D. Calais is $51^{\circ} 39' 50''$: also that between Gravelines and Calais is $56^{\circ} 42' 0''$. Now the difference between these two last $4^{\circ} 47' 50''$ being added to $72^{\circ} 11' 48''$, we shall have $76^{\circ} 59' 38''$. and its complement $103^{\circ} 0' 22''$, for the angles that the meridian of Dunkirk makes with the line drawn from thence through the point M to Calais: to which last angle adding

the former convergence $1' 50''\frac{1}{2}$, we have $103^{\circ} 2' 12''\frac{1}{2}$ for the angle that the meridian of Paris produced northward from M makes with the said line; but by the former set of angles it was found to be only $103^{\circ} 0' 57''\frac{1}{2}$, wherefore the difference is $1' 15''$.

From M. CASSINI's book it appears, that Dunkirk is north from Paris 125515.25 toises, which make 133768 fathoms; and the point M being south from the tower of Dunkirk 351 fathoms, there remains for the distance of M northward from the Royal Observatory 133417 fathoms. Now, with this distance as radius, the value of an angle of $1' 15''$ is $48\frac{1}{2}$ fathoms, equal to $4'' 34'''$ of longitude. Thus the point M, instead of being westward from Dunkirk 1514 fathoms, will, by the last set of angles, only be removed from it $1465\frac{1}{2}$ fathoms: wherefore the difference between the mean and extreme places of M, in this way of considering it, will amount to $24\frac{1}{4}$ fathoms, about four times as much as that resulting from the comparison stated in the 57th page. In the parallel of Greenwich the extreme difference will amount to 58.4 fathoms, or about $5\frac{1}{2}$ seconds of longitude, not much more than one-third part of a second of time.

In this sort of uncertainty, with regard to the precise point of intersection of the meridian of the Royal Observatory of Paris with the line joining Dunkirk and Calais, the only thing that can be done on our part, is to consider the mean position of M as just, that is to say, to suppose it to be 1514 fathoms westward from the great tower of Dunkirk, and having connected it with the British triangles, to shew then what angle its meridian will make with the line drawn from Dunkirk to Calais.

Comparison

Comparison of the celestial arc of the meridian, comprehended between the parallels of Greenwich and Perpignan, with the corresponding portions, measured and computed, of the terrestrial arc of the said meridian, between M and Perpignan.

In the consideration of this matter it is to be observed, that M. CASSINI has divided the celestial arc between the parallel of Dunkirk, or, which is the same thing, between the parallel of M and Perpignan, into four principal sections; *viz.* that from M to Paris, from Paris to Bourges, from Bourges to Rodés, and from Rodés to Perpignan; affigning to each section the measured portion of the corresponding terrestrial arc, resulting from the triangles of the meridian.

By pages 110, 111, 112, of the first part of the book it appears, from the mean observed zenith distances of four stars, that the arc of the heavens, between the parallels of Dunkirk and Perpignan, contains

- - - - -	8 20 2 26
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And from the mean of the observations of a like number of stars, the parallel of Rodés is distant from that of Dunkirk,

-	6 50 51 14
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Wherefore Rodés is north from Perpignan

-	1 39 11 12
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Bourges, by the mean zenith distances of two stars, is south from Dunkirk,

- - - - -	3 56 59 55
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Wherefore Bourges is north from Perpignan,

-	4 23 2 31
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And by another mean, in p. 112. it is

- -	4 23 2 35
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The mean of which two means gives for the distance of their parallels,

- - - - -	4 23 2 33
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Perpignan is south from the Royal Observatory at Paris, by the	°	'	"	'''
mean zenith distances of four stars,	6	8	11	52
Hence Paris is south from Dunkirk	2	11	50	34
Rodés is south from Paris, by the mean zenith distances of two				
stars	4	28	59	45
Hence Perpignan is south from Rodés,	1	39	12	7
And by a former result it was found to be,	1	39	11	12
Hence the mean of the two means gives, for the distance of the				
parallels of Rodés and Perpignan,	1	39	11	28

The tower of the great church at Dunkirk is northward from				
what was the station of the sector $84\frac{1}{4}$ toises, equal to 89.8 fathoms,				
which correspond to an arc in the heavens of	0	0	5	19
Wherefore the tower is north from Paris,	2	11	55	53
But the point M is south from the tower 351 fathoms, which				
answer to an arc in the heavens of	0	0	20	45
Hence the point M is northward from the Royal Observatory at				
Paris,	2	11	35	8

Now, from these *data*, together with the latitude of the Royal Observatory at Greenwich $51^{\circ} 28' 40''$, and that of Paris $48^{\circ} 50' 14''$, we shall have the latitudes of the several stations between Greenwich and Perpignan, with their differences, or the celestial arcs comprehended between them, as underneath.

Stations.	Latitudes.	Diff. or celestial arcs.
Greenwich Royal Observatory,	$51^{\circ} 28' 40''$	$0^{\circ} 26' 50''$
Point M near Dunkirk,	$51^{\circ} 1' 49''$	$2^{\circ} 11' 35''$
Paris Royal Observatory,	$48^{\circ} 50' 14''$	$1^{\circ} 45' 9''$
Bourges,	$47^{\circ} 5' 44''$	$2^{\circ} 43' 51''$
Rodés	$44^{\circ} 21' 13''$	$1^{\circ} 39' 11''$
Perpignan (<i>St. Faumes</i>)	$42^{\circ} 42' 2''$	

This latitude of Perpignan $42^{\circ} 42' 2'' 8'''$ is what results from the immediate comparison of the lengths of the celestial arcs,

arcs, as determined by the zenith distances of stars, taken with a sector of six feet radius, and where the observations are so nearly consistent among themselves, as to leave little doubt of their accuracy; but in the 290th page of M. CASSINI's book, so often quoted, as well as in the 170th page of his *Description Géographique de la France*, published in 1783, the latitude of Perpignan is given $42^{\circ} 41' 55''$, which is $7'' 8'''$ less than that deduced from the observations, without any reason that I can perceive being assigned for the reduction.

Perpignan, the southernmost station of the meridian line extending from Dunkirk through the whole Kingdom of France, is situated at no great distance from the bottom of the Pyrenean mountains, where that lofty range ends at the Mediterranean sea. M. DE LA CAILLE was of opinion, that the plummet of the sector must have been affected by the attraction which it would suffer from that cause; a supposition which, nevertheless, has been doubted, since the observations made in this country on the attraction of *Schehallion*: for by these it appeared, that the effect, although sensible, was but small, even when the sector was placed as near as possible to the opposite sides of the mountain. It is indeed true, that the *Canigou*, the highest of the Pyrenean range, being situated obliquely to the meridian, and at a considerable distance from Perpignan, would not probably occasion much deviation in the plummet; yet, on the other hand, when we compare the very trifling quantity of matter in *Schehallion* with the immensity of the mass in the Pyrenees, in the direction of the meridian, I cannot help being of M. DE LA CAILLE's opinion, that the plummet of the sector would be sensibly affected, that is to say, it would be drawn to the southward out of its perpendicular direction, and would thereby give the zenith distance of the pole,

pole, or any other northern star, too little, and consequently a latitude too great. Until triangles shall have been extended beyond the Pyrenees, and the sector placed on the south side of the range, the quantity of this attraction (by its double or counter-effect) cannot possibly be ascertained. I will, however, only suppose it to have been $10'' 8'''$ to be deducted from the latitude of Perpignan, which will then become $42^{\circ} 41' 52''$, only three seconds less than that assigned to it in M. CASSINI's two books before mentioned. Thus the arc between Rodés and Perpignan will be $1^{\circ} 39' 21'' 36'''$, and the total celestial arc between Greenwich and Perpignan will be $8^{\circ} 46' 48''$, as may be seen by attending to the four columns towards the left-hand of the annexed table of comparison.

With regard to the corresponding terrestrial arc, under which head are arranged the eleven columns towards the right-hand of the table, it is to be observed, that various measurements have at different times been made in different latitudes of the lengths of the degrees of the meridian, for the purpose of obtaining, within certain limits at least, the true figure and dimensions of the earth. The most essential operations of this sort, as having been executed with most care, with the best instruments, and at the greatest distances from each other, have all been done within these last forty or fifty years; namely, in Peru under the equator, in middle latitudes in France and Italy, and in Lapland near the polar circle. The attraction of mountains, and unavoidable errors in the execution, will ever prevent just conclusions from being drawn from the comparison of measurements made too near each other. These last will always be found to differ more or less among themselves. Sometimes even the results may become absurd or contradictory. In cases of this sort, a mean of several should no doubt be

taken for a mean latitude, unless there should be sufficient grounds for rejecting any from the number, as differing too much from the others. Hence it is, that philosophers are not yet agreed in opinion with regard to the figure of the earth; some contending, that it has no regular figure, that is to say, not such as would be generated by the revolution of a curve around its axis. Others have supposed it to be an ellipsoid; regular, if both polar sides should have the same degree of flatness; but irregular, if one should be flatter than the other. And, lastly, some suppose it to be a spheroid differing from the ellipsoid, but such as would be formed, nevertheless, by the revolution of a curve around its axis; although in this case too one polar side may not be similar to, but more or less oblate than the other.

In order, therefore, to put this matter in its true light, and to enable every one to judge, by simple inspection only, which of the theories agrees best with actual measurement, I have computed on ten different *hypotheses*, and arranged in their order, the lengths of the arc between Greenwich and Perpignan; as also some other chief properties of each figure, which last fill up the space towards the bottom of the table. This mode of collecting the results seemed to me to be the most distinct that could be followed, to avoid that perplexity which must for ever occur in referring back to, and comparing, many numbers together, computed on different systems, when the whole are not placed before the eye at once.

The first of the eleven columns, or that which comes next to the celestial arc, contains the measured portions of the corresponding terrestrial arc, as far as they have yet been executed. The blanks at top cannot be supplied, until we shall

have determined the length MP, in the map, being the space comprehended between the parallels of M and Greenwich.

In the second column are arranged the computed dimensions appertaining to the earth as a sphere, supposing its semi-diameter to be a mean between the longest and shortest of M. BOUGUER's second spheroid. It is from the magnitude of this sphere that I compute the degrees of a great circle for the sides of spherical triangles. By adverting to the errors or differences between the measurement and computation, in their respective places, it will obviously appear that the earth differs very considerably from a sphere: for although the arc M Perpignan of $8^{\circ}\frac{1}{3}$ only exceeds the truth by 609 fathoms; yet an arc of equal length at the equator, *viz.* $8^{\circ}.33 \times 374.6$, would give an excess of 3120 fathoms; and at the polar circle $8^{\circ}33 \times 335.2$ would give a defect of 2792 fathoms.

After the sphere follow seven ellipsoids of different degrees of oblateness, from the first, whose semi-diameters have to each other the ratio of 179.047 to 178.047, to the seventh, where it is only that of 540 to 539. On the principles which have served as the foundation of the first and second, it will be necessary to make some remarks; but as to the others, a few words will suffice for each.

With regard to the first ellipsoid, supposing the earth to be homogeneous, it is well known, that the ratio of its semi-diameters may be found, by comparing with each other the lengths of the pendulums that vibrate seconds in different latitudes; which lengths are deduced from the seconds of acceleration, that the pendulum, so adjusted, and unalterably fixed as to length, at the equator, would perform in 24 hours, on being successively transported to different latitudes, as far as the pole, where the force of gravity being the greatest, the acceleration would likewise be the greatest. The calcu-

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lations for this purpose were first made soon after * Lord MULGRAVE's return from his Voyage towards the North Pole in 1773.

* The most northern experiments hitherto made with the pendulum, are those at Spitzbergen, in latitude $79^{\circ} 50'$, whereof an account has been given in the Voyage towards the North Pole in 1773. The machine made use of on that occasion belonged originally to the celebrated watch-maker, Mr. GRAHAM, and was lent for the purpose by its present proprietor, Mr. CUMMING, who has since obligingly permitted it to remain for several years, at two different times, in my possession, where it now is. Soon after the return from Spitzbergen, I ascertained its rate of going by my observatory clock, for a great while together, in very different temperatures; and thereby found, that the variation from heat and cold, namely, $\frac{6}{10}$ ths of a second for each degree of FAHRENHEIT, was considerably more than had been allowed for it, in determining the acceleration from London to Spitzbergen. About this time likewise, Dr HORSLEY discovered that an error had been committed by the astronomer (the late Mr. ISRAEL LYONS) employed by the Board of Longitude on that Voyage, in calculating the spherical triangle for the correction of the time between the 16th and 17th of July, on account of the obliquity of the transit-instrument. From Dr. HORSLEY's printed letter on this subject in 1774, it appears, that the difference from that cause amounted to 37 seconds in time, as given by the observation with the instrument and by the watch. And since there was reason to suspect, that the telescope had by accident been moved (one could not tell how much), it was judged safest to adhere to the acceleration $71''.08$ as given by the watch. The correction for the greater contraction of the steel rod, I found to be $2''.58$ to be subtracted, wherefore the acceleration from London to Spitzbergen became $68''.5$, and that from the equator to London being $156''$, the total acceleration from the equator to Spitzbergen consequently was $224''.5$. Now, supposing the length of the pendulum at the equator to be, as M. BOUGUER made it, just 38.9949 inches, we shall have its length at Spitzbergen 39.1978 inches; and thence the ratio of the semi-diameters of the earth, considered as an homogeneous ellipsoid, will be that of 193.1 to 192.1, instead of 183.7 to 182.7, which the acceleration $72''.7$ uncorrected would have given. And here it seems necessary to mention some other mistakes of computation, inadvertently fallen into by the astronomer, in deducing the ratios of the semi-diameters of the earth, as stated in the 179th page of the Voyage to the North Pole, not hitherto noticed that I know of.

1773. I am aware that experiments with the pendulum have not yet been made with that accuracy that the delicacy of their nature

With the accelerations, indicated in that page as arising from different experiments, or depending on the systems of different philosophers, the length of the Spitzbergen pendulum, and consequently the ratios, should have stood as in the annexed table, with which it will be perceived they differ very considerably. Sir ISAAC NEWTON's ratio is placed last, as probably differing most from the truth.

Experiments.	Acceleration			Length of the Spitzberg. pendulum.	Ratio of the semi diameters of the earth.
	From the equator to London.	From London to Spitzberg.	Total.		
Said to be, but erroneously, Sir I. NEWTON's	} 156" {	+ 66.9	222.9	In. 39.1964	194.5 to 193.5
Mr. CAMPBELL's		+ 76.6	232.6	39.2052	186.5 to 185.5
M. MAUPERTUIS's		+ 86.5	242.5	39.2141	178.8 to 177.8
Ld MULGRAVE's correct.		+ 68.5	224.5	39.1978	193.1 to 192.1
Sir ISAAC NEWTON's		+ 32.4	188.4	39.1652	230. to 229.

In this manner the lengths of the pendulum having been found for all the latitudes where the best or most consistent experiments had been made on its acceleration, and these lengths having been successively compared with each other from Spitzbergen to the equator, 119 results in the whole were produced. But as this number comprehended the comparisons of those at the Cape of Good Hope and the Isle of France, both in fourth latitude, these being thrown out, as well as the Porto Bello pendulum, and some few others more irregular than the rest, there remained at last 75 results, the arithmetical mean of which gave the ratio 179.047 to 178.047, as mentioned in the text.

In case Mr. CUMMING's machine should at any time hereafter be employed in the same sort of experiments, it may be proper to observe, that the diameter of the brass ball is 3.906 inches, and its weight 63726 Troy grains. The weight of its bulk of mercury is 106980 Troy grains. Hence the weight of mercury is to the weight of this ball of brass, in air of the heat of 62° of FAHRENHEIT, as 1.678752 to 1; and the weight of this brass to air is as 7673 to 1. The experiments for this purpose, in which Dr. GEORGE FORDYCE assisted, were made in the house of Mr. ALCHORNE, at his Majesty's Mint in the Tower, on the

nature seems to require, and which at some future period should, and probably will, be undertaken on a more extended plan, with the very best machines that can be constructed for the purpose. Many ages may elapse before measurements of any considerable portion of the surface of the earth can be made in very high southern latitudes, so as to determine, with any tolerable degree of exactness, whether the southern semi-spheroid be similar to the northern, supposing both to be figures of revolution. But by good experiments with the pendulum alone, easily repeated in all latitudes, the ratio of the semi-diameter of the equator to the semi-axis on both sides may be readily obtained. In the mean time I thought it might be useful to shew the result of a comparison of the most consistent experiments of that sort, that have hitherto been made in different latitudes, after having applied to those at Spitzbergen certain corrections, which seemed necessary, as further explained in the note. Thus it appears, that the arithmetical mean of 75 comparisons between Spitzbergen and the equator gives the

the 8th of April and 6th of June, 1776; and the beam made use of, with 15 lbs. weight in each scale, was true to three grains.

The machine, in its present state, although better fitted for experiments than it was originally, since it has a wheel added, whereby it registers its own time, and now goes for 12 hours without winding up, is no longer the same that performed at Spitzbergen: for in the interim of the two times it has been in my possession, the steel rod having by accident been broken, Mr. CUMMING substituted another, as like to the former as possible with regard to size, but which will, in all probability, be susceptible of a different expansion and contraction from the effects of heat and cold. In the application of the machine, I have usually loaded the top of it with about 24 lbs. weight of lead, in order to render the center of suspension as little liable to motion as possible: yet, notwithstanding this precaution, a pointed plummet, suspended to the top of the frame, has a small degree of counter-vibration to that of the ball, which no doubt must produce some effect.

ratio

ratio of the semi-diameters formerly mentioned 179.047 to 178.047. On this hypothesis the arc MP should contain 27350 fathoms. The error on the total arc M Perpignan amounts to 2078 fathoms. M. BOUGUER'S degree at the equator being adhered to, the 45th of latitude will exceed the truth 216, and that at the equator 148 fathoms,

The ratio of the semi-diameters of the second ellipsoid, which comes now to be spoken of, has been obtained by the comparison of such measured lengths of the degrees of the meridian in different latitudes, as have been found to be most consistent with each other. Our countryman, Mr. NORWOOD, was the first, of late times, who made any attempt of this sort. But the measurement, executed by him in the year 1633, between London and York, has no pretence to exactness, since he himself tells us, *that when he did not measure, he paced!* Besides, his degree is as great, or even greater, than that in Lapland; and these are surely sufficient reasons for rejecting it from the comparison. The degree measured by M. LIESGANIG in latitude $45^{\circ} 57'$, in that part of Poland lately fallen to the share of the Emperor and annexed to Hungary, being so much shorter than degrees to the southward of it, gives grounds to suspect, that some error had crept into that operation, or that the plummet had been affected by the attraction of neighbouring mountains, and therefore is not made use of on the present occasion. M. DE LA CAILLE'S degree at the Cape of Good Hope, being in south latitude, and so much greater than those of the same height in northern latitudes, is improper likewise to be brought into the comparison, lest the difference may have arisen from a dissimilarity in the two polar sides of the ellipsoid. The degree measured in the north of France, compared with that in Austria, coming out
 absurd,

abfurd, it has been judged beft to take a mean between them for a mean latitude. In like manner the latitudes of the two Italian degrees differing but little from each other, a mean length has been taken between them for a mean latitude. Accordingly the latitudes and the meafured lengths of the degrees which, in the fecond ellipfoid, have been compared together, will appear as underneath :

Obfervers names.	Countries.	Latitudes.		Meafured lengths.	
BOUGUER,	Peru,	-	-	0 0	- 60484.5
MASON and DIXON,	Maryland,	-	-	39 12	- 60628.5
BOSCOWICH,	Italy,	-	43° 0'	} 43 52 {	{ 60725.5 } { 60821.3 }
BECCARIA,	Piedmont	-	44 44		
CASSINI, &c.	Middle of France,	-	45 0	-	60777.6
LIESGANIG,	Auftria,	-	48 43	} 49 3 {	{ 60839.4 } { 60826.6 }
CASSINI, &c.	North of France,	-	49 23		
MAUPERTUIS, &c.	Lapland,	-	66 20	-	61194.3

Now thefe fix degrees, being fucceffively compared with each other, fifteen refults are thereby obtained, whereof the arithmetical mean gives for the ratio of the femi-diameters of the ellipfoid that of 192.483 to 191.483. By adverting to the table, it will further appear, that the arc MP fhould be in length 27331 fathoms. The arc M Perpignan exceeds the truth 1758, the 45th of latitude 180, and that at the polar circle near 88 fathoms.

The ratio of the femi-diameters of the third ellipfoid 216.06 to 215.06 is obtained by adhering to the meafured lengths of the degrees at the equator and polar circle. According to this hypothefis the arc MP fhould contain 27301 fathoms. The arc M Perpignan exceeds the truth 1288, and that at the 45th of latitude more than 128 fathoms.

The

The ratio of the semi-diameters of the fourth ellipsoid 222.55 to 221.55 is the same, as may be seen by referring to the table, with that assigned by M. BOUGUER to his first spheroid, where the increments to the degrees of the meridian above that at the equator are as the second power or squares of the sines of the latitudes. It was intended chiefly to shew how small the difference is between the magnitudes and nature of the curves of the two figures. The arc MP should contain 27294 fathoms. The arc M Perpignan errs in excess 1177 fathoms. The 45th degree exceeds the truth 116 fathoms; and that at the polar circle falls short of the measured length 21 fathoms: M. BOUGUER's degree at the equator being adhered to as the standard.

The ratio of the semi-diameters of the fifth ellipsoid, 230 to 229, is that assigned to the earth by Sir ISAAC NEWTON. On this hypothesis the arc MP should contain 27241 fathoms. The arc M Perpignan only exceeds the truth 202 fathoms, because the 45th degree of the meridian is here adhered to as the standard length. But then the degree at the equator falls short of the measurement 102 fathoms, and that at the polar circle $146\frac{1}{2}$; wherefore, an arc of $8^{\circ}\frac{1}{3}$, in the first case, would be defective 850, and in the last 1220 fathoms.

The ratio of the semi-diameters of the sixth ellipsoid, 310.3 to 309.3, is obtained by adhering to the measured lengths of the degrees at the equator and 45th of latitude. The arc MP should contain 27230 fathoms. The arc M Perpignan only exceeds the truth 131 fathoms; but on this hypothesis, the degree at the polar circle would be defective near 217 fathoms, and consequently on $8^{\circ}\frac{1}{3}$ the error would be 1807 fathoms.

The seventh or last ellipsoid, being that of the least flattening, has for the ratio of its semi-diameters 540 to 539. The arc MP should contain 27206 fathoms. The 45th degree of latitude being adhered to as the standard, the arc M Perpignan would only exceed the truth by 46 fathoms; but, on the other hand, the degree at the equator erring in excess $124\frac{1}{2}$ fathoms, and that at the polar circle being defective near 303; therefore, in the first case, the error on $8^{\circ}\frac{1}{3}$ would be 1037, and in the last 2524 fathoms. Hence it is obvious, that the arcs of an ellipsoid, however great or small the degree of its oblateness may be, will not any way correspond with the measured portions of the surface of the earth: for if we retain the length of M. BOUGUER's degree at the equator as the standard, and make the ellipsoid extremely flat, as in N^o 1. the figure will become too prominent in middle latitudes, that is to say, the curve will rise above the real surface of the earth, and, in proportion to the excess of the radius, will always give degrees that exceed the measured length. On the contrary, if we give the ellipsoid a small degree of flatness, as in N^o 7. and adopt the measured length of the 45th degree as the standard, the measured and computed arcs will nearly agree in middle latitudes; but at the equator the curve will rise very considerably above the surface, and will thereby give degrees that are too great; while at the polar circle it will fall below it, and give degrees that are too little in the proportion of about $2\frac{1}{2}$ to 1 compared with the error at the equator. From all which we may conclude, that the earth is not an ellipsoid.

The two columns towards the right-hand of the table, contain the arcs of two spheroids differing from the ellipsoid. The first is that adopted by M. BOUGUER as his first hypothesis, where the increments to the degrees of the meridian

above that at the equator follow the ratio of the second power or squares of the sines of the latitudes, and to which he has suited his first table of degrees, N° 32. p. 298. This spheroid differs but insensibly, as has been already mentioned, from the fourth ellipsoid. They have both the same semi-diameters; but the arcs of the spheroid being somewhat longer than those of the ellipsoid, the former thereby becomes, in a trifling degree, more prominent in middle latitudes. On this hypothesis the arc MP should be in length 27295 fathoms; M Perpignan exceeds the measurement 1196 fathoms; and the degree at the equator being adhered to as the standard, the 45th errs in excess 118, while that at the polar circle is defective only 20 fathoms.

The second spheroid is that whereon M. BOUGUER founded his second hypothesis, which supposes the increments to the degrees of the meridian, above that at the equator, to follow the ratio of the fourth power or squared squares of the sines of the latitudes, and to which he has adapted his second table of degrees N° 38. p. 305. It will be perceived, that the ratio of the semi-diameters of this spheroid, *viz.* 179.4 to 178.4 differs little from that appertaining to the first ellipsoid; but here the curve falling considerably within, that is to say, being less prominent than the ellipsoid in middle latitudes, the arcs are thereby contracted in such a manner as to agree within 5 fathoms with the measured length of the meridian of France, in an extent of about $8^{\circ}\frac{1}{3}$, comprehended between M near Dunkirk, and Perpignan situated at the bottom of the Pyrenean mountains. By inspection of the table it will further appear, that the errors in the several sections of this arc are not only small, but they are sometimes *plus* and sometimes *minus*, a never failing proof that, as far as our present *data* will enable

us to judge, the figure here assigned to the earth, notwithstanding what has been alledged to the * contrary, is exceedingly near the truth. According to this hypothesis, the distance MP on the meridian of Paris, which is yet to be determined by our trigonometrical operations, should contain 27243 fathoms, being only 35 fathoms less than what is given by the mean of the seven different ellipsoids, a space not amounting quite to 2'' of latitude. The result of the measurement of this space, answering to an arc in the heavens of 26' 50'' 52''' of latitude, will be a further confirmation, or otherwise, of the justness of the theory. The degree at the equator being adhered to as the standard, it will be seen from the table, that the 45th is defective 37.6, while that at the polar circle errs in excess 9.4 fathoms.

* Mr. J. KLOSTERMANN, Inspector of the Corps of Pages at St. Petersburg, in his manuscript Memoir, some time since transmitted to the Royal Society, to the Academy of Sciences at Paris, and also to that at Gottingen, has endeavoured to shew, that the French trigonometrical operations are extremely erroneous. It would seem, nevertheless, that he has attempted to prove too much. He should certainly have confined his criticism to the triangles of the meridian only, which are distinguished from the others in M. CASSINI's book, by being printed in larger characters, without drawing conclusions from very acute angles, which, although inserted in the general register, were not made use of in the determination in question. But as the Royal Academy of Sciences will, no doubt, vindicate the credit of their own operations, I shall only further remark, on the Literary News from Gottingen (*Nachricht aus den Göttingischen Anzeigen von gelehrten Sachen*, 117 Stück, 1785), which accompanied the said Memoir, and where it is said, "That M. BOUGUER's hypothesis of the 4th power fell to the "ground, so soon as other degrees were measured than those on which he had "founded it," that I confess myself to be quite of a different opinion, not doubting, that when the comparison is fairly drawn between this and every other system that has hitherto been submitted to the consideration of the public, M. BOUGUER's will be found to be justly entitled to the preference, which I have here endeavoured to give it. His works shew, that he was a man of very superior abilities, eminent as a mathematician, and perhaps the best practical one that ever existed.

Besides these two spheroids of M. BOUGUER, I had computed the arcs of another between the two, where the increments to the degrees of the meridian were in the ratio of the fractional power $3\frac{4}{100}$ of the sines of the latitudes. Thus the computed agreed accurately with the measured dimensions in three principal parts of the earth's circumference, namely, at the equator, 45° and $66^\circ 20'$ of latitude, at the same time that the arc MP contained 27257 fathoms; but the labour of computation being hereby greatly augmented, and the error on the total arc between M and Perpignan amounting to 290 fathoms in excess, this system did not seem to me to deserve to be put in competition with the simplicity of that of M. BOUGUER, who indeed, for the same reason, preferred the fourth to the fractional power $3\frac{1}{11}$ ths, which he tells us was that which still came nearer the truth. In short, it must be from the results of future operations executed in very high latitudes, and the measurement of degrees of longitude on the equator, that we can hope to have sufficient authority for any correction or a melioration of the system of M. BOUGUER.

Differences of longitude.

Hitherto there has been no particular reference to some lines at the bottom of the table, containing the computed lengths of degrees of longitude on each hypothesis, in three different latitudes, namely, the equator, $43^\circ 32''$ and $51^\circ 28' 40''$. No measurements of degrees of longitude, as far as I know, have ever been executed with sufficient care and accuracy, except that in the south of France, as mentioned in the 105th and 106th pages of M. CASSINI's book, which was determined by the

the repeated explosions of gunpowder in the open air, and found to contain 41618 toises, equal to 44354.4 fathoms. By attending to the table it will be seen, that the error in excess of M. BOUGUER's theory, on the length of this degree trigonometrically measured, amounts only to 19 fathoms, which is little more than $\frac{1}{10}$ th part of a second of time.

In fixed Observatories, where able astronomers have been for many years employed in repeating their observations of the heavenly bodies, it seems surprising, that any doubt should remain with regard to what is called the astronomical difference of longitude, or, in other words, the difference of time between them; yet it has been alledged, that an uncertainty of this sort exists, even with regard to the situation of Greenwich and Paris, which, reckoned by its extremes, extends to about 10 or 11 seconds, answering in the latitude of Greenwich to the enormous difference in space of between 1600 and 1700 fathoms! But it will be considered as still more wonderful, if between two British Observatories, Greenwich and Oxford, which have been long supplied with great and costly instruments of the very best kinds, there should remain an uncertainty in this respect of 2 or 3 seconds of time: for in the latitude of Greenwich 3 seconds correspond to 477, and in that of Oxford to 474 $\frac{1}{2}$ fathoms. These, however, are points which must be left to the respective astronomers to settle in the best way they can; and it is not to be doubted, that the Astronomer Royal will throw a new and very satisfactory light on the matter, in the Paper which he proposes about this time to lay before the Royal Society, along with M. CASSINI's Memoir, which, for that purpose, has now been nearly two years in his possession.

With

With regard to the trigonometrical operation (which may be considered as infallible, because, by means of the base of verification, it will prove itself, and if small errors unavoidably arise in the course of a long suite of triangles the maximum of these may be always ascertained), I have no doubt that the distance between Greenwich and the point P in the map may thereby be determined to a very small number of fathoms, perhaps to fifteen or sixteen on a difference of longitude of about $2^{\circ} 20' 20''$, and therefore to about $\frac{1}{25}$ th part of a second of time on each degree. This, for any useful purpose, will certainly be admitted to be sufficiently near the truth, and is probably considerably nearer than it will be brought for many years to come, by a mean of the best observations of the heavenly bodies, if these should be found in the present state of the matter to leave it yet doubtful to two or three seconds.

The astronomical difference of time may likewise be obtained by experiments on the instantaneous explosion of light; but these I would propose to be made subsequently to the trigonometrical operations. The station of *Tatterlees*, towards the eastern extremity of our range of chalk hills, or some point near it, would seem to be the most proper for the place of explosion, because it can be seen from *Bottle-hill*, on the same range, and nearly in the meridian of Greenwich Observatory. It is not to be doubted, that *Tatterlees* may be seen from *Fienne Windmill*, or even perhaps from that of the *Brunenberg*; since they are both situations, on the continuation of the same range in France, the distance being shorter too, and little land, but chiefly sea, intervening. Let us then suppose, that the two astronomers with their clocks and transit-instruments are posted, one at *Bottle-hill*, and the other at the *Brunenberg*, while gunpowder is repeatedly exploded at *Tatterlees*, or while the Indian lights

lights are alternately exhibited, and again covered by an extinguisher prepared for the purpose, which operation may be repeated several times the same evening; it is certain, that a just mean being taken between the instants so marked by the respective clocks, well regulated before-hand, the difference of time between the two extreme stations will thereby be obtained to a very considerable degree of accuracy, and probably more to be relied upon than that resulting from the comparison of the observations of the heavenly bodies.

Rockets have been proposed by some to be applied in this way; but even the largest fort, carrying up too small a body of white light, could, I think, only be depended upon for short distances; and since the actual error of observation would be the same on a short as on a long distance, remote stations only should be chosen for conclusive experiments of this nature. It is imagined, that by means of a balloon, a set of light-balls might be sent up of such a volume as to be seen, from their great elevation in the air, at stations very remote from the place of explosion. The balloon would ascend with a burning fuze attached to the first ball, and having attained the wished-for height, it might be made fast below, by means of a cord fixed to it for the purpose*. The first ball, on its explosion, would communicate fire to the second, and so in succession, at intervals of time proportionable to the lengths of

* It is to be understood, that the practicability of fastening the balloon to the earth by means of a rope must be previously ascertained: for if that should be found impossible, without the wind forcing it down again to the ground, then it must be sent up detached with burning fuzes fixed to it, of still greater dimensions. By means of these it is imagined, that the track of the balloon in the air might be pointed out to the remote observers, who might perhaps be better enabled thereby to watch for and distinguish the successive instants of the bursting of the white lights.

the fuzes. Thus 8 or 10 instants might be marked from the experiment of a single balloon, which might then be hauled down to be reloaded for a repetition. But whatever might be the mode adopted as the best for conducting experiments of this nature, the observers must not only be very attentive and diligent, but also quick-sighted, have their clocks nicely regulated indeed, and the trials must be many times repeated before the uncertainty, even in this way, which seems to be the best mode, could be reduced to less than $\frac{1}{20}$ th part of a second of time, to which it may infallibly be brought by trigonometry.

Having in this manner shewn what probable degree of exactness may be expected in the various, but usual, ways of ascertaining the difference of longitude between the Observatories of Greenwich and Paris, and compared the results with the uncertainty that seems yet to exist in this matter from the state of astronomical observations; let us next see how Mr. RAMSDEN's instrument is likely to perform, when actually applied to the determination in question, by the observed angle between the pole star in its eastern or western azimuth, and a very remote station, whose distance from the instrument is known by the series of triangles, and distinguishable by the Indian lights at night, for the purpose of this particular observation.

With an instrument, carrying telescopes so good that the pole star may be seen in daylight, it is obvious, that the bisected angle between the star in its eastern and western azimuths will give at once the polar distance of the star, and the true meridian of the place, as referred to any known stations visible at the time of observation. But as cloudy weather may often prevent a complete observation of this sort from being obtained, and since much time might be lost in attempting

attempting it, therefore the declination of the star settled for any particular period being accurately known*, its apparent distance from the pole may, by the established rules, be readily computed for any proposed day, as well as the precise times of its greatest elongations, twice in 24 hours, when in its eastern and western azimuths, at which times it will, for several minutes, appear, as to sense, stationary or without motion, except in altitude. These are, therefore, the best times for taking the angle between the star and any particular station, since the observations may be repeated frequently in the space of a few minutes, or until it shall be perceived that the star has again approached towards the pole. Now, suppose the station of the instrument to be at *Tatterlees*, whose distance from the perpendicular to the meridian of Greenwich, and consequently from its parallel, is known by the trigonometrical operation. The latitude of the station becomes known likewise; and let the co-latitude be $38^{\circ} 54' 20''$. Let us likewise suppose the distance of *Bottle-hill* on one side to be 44100 fathoms, equal to $43' 28''.6$ of a great circle; and that of the *Brunenberg* on the other to be 38250 fathoms, equal to $37' 42''.6$ of a great circle; and further, that on these two stations the Indian lights are exhibited for the time proposed. Now, let the angle between the meridian and *Bottle-hill*, and that between it and the *Brunenberg*, be observed by means of the pole star corrected for its distance for the day; and suppose the first to be $75^{\circ} 10'$,

* Since this Paper was written, the Astronomer Royal has been so obliging as to furnish me with the mean distance of the pole star from the pole, as settled at Greenwich by eight observations above and nine below it, made in the year 1786; whereby it appears, that the mean distance, reduced to the beginning of that year, was $1^{\circ} 50' 8''.35$; and the mean annual precession in declination being $19''.55$, consequently, the mean distance for the 1st of January, 1787, was $1^{\circ} 49' 8''.8$.

and the last $125^{\circ} 5'$; thus we shall have two spherical triangles to compute, in each of which two sides, and the contained angle, are known, and one side, *viz.* the co-latitude, is common to both. Now, from these *data*, making use of the half sum and half difference of the sides, we shall have the angles in these two triangles as * underneath, and the angle of longitude between *Bottle-hill* and the *Brunemberg*, equal to that at the pole, will be found to be $1^{\circ} 55' 56''.1$. If from this angle we deduct about $30''$ or $35''$, for the space that the *Bottle-hill* seems to be to the westward of the meridian of Greenwich, there will then remain $1^{\circ} 55' 21''$ for the east longitude of the *Brunemberg*, being very nearly that expressed in the map of the triangles which accompanies this paper.

* Bottle-hill.			Brunemberg.		
Half difference	-	$51^{\circ} 25' 14''.05$	Half difference	-	$26^{\circ} 44' 12''.2$
Half sum	-	$52^{\circ} 32' 25''.07$	Half sum	-	$27^{\circ} 32' 57''.3$
Angle at Bottle-hill		$103^{\circ} 57' 39''.12$	Angle at Brunemberg		$54^{\circ} 17' 9''.5$
Angle at the pole	-	$1^{\circ} 7' 11''.02$	Angle at the pole	-	$48^{\circ} 45''.1$
Contained angle	-	$75^{\circ} 10' 0''.$	Contained angle	-	$125^{\circ} 5' 0''.$
Sum of the three angles		$180^{\circ} 14' 50''.14$	Sum of the three angles		$180^{\circ} 10' 54''.6$
Angle of convergence		$52^{\circ} 20'.88$	Angle of convergence	-	$37^{\circ} 50'.5$
Excess above 180°	-	$14^{\circ} 50''.14$	Excess above 180°	-	$10^{\circ} 54''.6$
Angle of longitude	-	$1^{\circ} 7' 11''.02$	Angle of longitude	-	$48^{\circ} 45''.1$
Sum of the two longitudes $1^{\circ} 55' 56''.12$.					

It is to be observed, that the meridians, which are all parallel to each other at the equator, on their departure from thence converge more and more as they approach towards the pole, where the angle of convergence becomes equal to the angle of longitude. It may also be remarked, that the angle of convergence, augmented by the excess of the three angles of the spherical triangle above 180° , is always equal to the angle of longitude, or that at the pole. And as this holds universally in all latitudes, it affords a ready means of proving that the computations are just.

As far as we are enabled to judge at present, from the examination of the divisions of Mr. RAMSDEN's instrument, there is every reason to believe, that in taking angles around the horizon, the mean of several repetitions of the same angle, as referred to different parts of the circumference of the circle, will differ very little from the truth, so little indeed, that in many cases the error will totally vanish. But in elevating the telescope towards the pole, let us suppose that an error of 5 seconds on each of the contained angles at *Tatterlees* has been committed; and further, that even an error of 5 seconds of latitude, equal to about $84\frac{1}{2}$ fathoms on the meridian, may have been fallen into, in estimating the co-latitude (which never can happen, but is only here admitted, to place the example in the most disadvantageous circumstances possible); then whoever will give themselves the trouble to recompute the two triangles with these new *data*, will find the result in longitude not to be varied thereby, in the first case above $\frac{1}{5}$ th part of a second, or $\frac{1}{75}$ th part of a second in time; and in the last not quite 1 second, or $\frac{1}{15}$ th part of a second in time. Hence I conclude, that the best mode of determining the differences of longitude will be by the instrument itself, applied in this way, in taking the angles between the pole star and very remote stations, distinguishable at night by the help of the Indian lights, and whose distance is accurately known. This method will, it is true, be liable, as well as astronomical observations, to the imperfections of the instrument, particularly those of the telescope, and the unavoidable error in its application; but, on the other hand, it will be entirely free from the irregularities of clocks, and the imperfections of vision in marking the instantaneous explosion of light. When both methods have been repeated a sufficient number of times, with all

imaginable care, we shall then, and not till then, be able to judge to which the preference may be due. Thus five or six long stations, in or nearly in the parallel of Greenwich, such, for instance, as that of Shooter's-hill Tower, would reach from the east quite to the west of the island: and as a very considerable degree of consistency might be expected among the results for equal portions of the parallel, this method seems to be as likely as any to furnish *data* for determining the nature of the spheroid or figure of the earth.

Table of the degrees of the earth, constructed on the hypothesis of
M. BOUGUER.

Beside the table of comparison of the arc between Greenwich and Perpignan, which I have already endeavoured to explain, this Paper is accompanied with another, which, as well as the former, was originally intended solely for my own use. With this view it was at first only computed for every five minutes of the 51st and 52d degrees of latitude, that I might thereby be enabled more readily to compare the longitudes and latitudes of the respective stations in our progress towards the coast; and more particularly to fit it for operations in the south parts of England, as likely to be first carried into execution. In order, however, to render it more generally useful, it has since been extended to every five degrees in the higher and lower parts of the quadrant, and to every single degree in intermediate latitudes. The table not only contains the degrees of the meridian and of longitude, but also those of a great circle perpendicular to the meridian, and likewise such as are oblique to it, for the other seven points of the compass. With regard to the construction of the table, it is only necessary to make
some

some remarks on that column of it, which contains the sum of the three equations, or difference between the degree of the meridian and the corresponding degree of the great circle perpendicular to it, which is the most troublesome to compute, but must be found before the degree of longitude can be obtained. The first part of M. BOUGUER's equation consists of $\frac{7}{13}$ ths of the difference between the degree of the meridian at the equator and that at the pole, viz. 545.12 fathoms, to be constantly added. Secondly, $\frac{1}{3}$ ths of the increment of the corresponding degree of the meridian above that at the equator, to be subtracted; and, thirdly, $\frac{4}{13}$ ths of a third proportional to the excess of the degree at the pole above that at the equator as radius, and the sine of the corresponding latitude, to be added. Now it will be found, that this last part of the equation $\frac{4}{13}$ ths, if uniformly applied, would have produced absurd results at the 75th degree of latitude, that is to say, the degrees of a great circle would there have become greater than the degree at the pole. The equation $\frac{4}{18}$ ths or $\frac{1}{4}$ th would in like manner have produced absurd results between the 79th and 80th degree. Even $\frac{2}{9}$ ths will not go on further than the 85th; and the highest equation that will go through the whole quadrant, uniformly applied, must not exceed $\frac{80}{370}$ parts of the third proportional. M. BOUGUER himself had found this, and accordingly had applied the equation with a certain modification or abatement, which nevertheless he makes no mention of in his book. Seeing, therefore, that the degrees of a great circle perpendicular to the meridian differ most from those of latitude about the tropics, I have, at the 20th degree, applied the equation $\frac{4}{13}$ ths or $\frac{80}{370}$ parts, and made the divisor increase by unity for each degree of the quadrant above that point to the pole, where it becomes $\frac{80}{370}$ parts. Below the 20th degree
the

the divisor, in like manner, increases by unity to the equator, where it becomes $\frac{80}{320}$ parts. With respect to the degrees of great circles, obliquely situated to the meridian, they were chiefly intended for facilitating the computations of curvature and refraction, where, in any particular case, it might become necessary to obtain, to great precision, the relative height of a very distant station with regard to that occupied by the instrument: for in such cases, in strictness, the angle which the line makes with the meridian should be attended to.

The computations * for the construction of this table have been, as may easily be judged, sufficiently laborious; but having once begun, I was induced to go through with them, conceiving that a table of this sort would be of general utility; and that it might be very long indeed before the results of future operations, yet to be undertaken in different and very remote parts of the globe, would furnish *data* for any thing better. It must be by the united efforts of enlightened nations, conspiring as it were together in promoting the cause of science, that great objects, such as the determination of the magnitude and figure of the earth, can ultimately be obtained. Each should contribute to it, and all have it more or less in their power, according to their particular situation, and that

* Much care has been bestowed in the various computations for the construction of the two tables appertaining to this Paper. In the last, which has been the most laborious, the differences seem to go on so uniformly, it is hoped, no error of any consequence will be found. Here it is proper that I should mention a typical *erratum* in one of the tables belonging to my Paper on the Barometer, published in the LXVII. volume of *Transactions*, for 1777. It is in Tab. VI. containing observations on heights near Carnarvon. In the column of observed height of Mercury on Moel Eilio, August 4, 1775, 1 h. 7 m. P. M. instead of 27.714 read 27.214.

portion of its surface which has fallen to their share in the general distribution of things.

Under the auspices, and immediately under the eye, of a Sovereign who loves and cherishes the sciences, much may be done within the limits of the British islands, which, reckoning from Eastness in Suffolk to the western parts of Kerry in Ireland, contain near 12 degrees of longitude; and counting from the Channel on the south to the Orkney islands only on the north about 9 degrees of latitude. With this general purpose in view, and at the same time to lay the best foundation for the survey of the British islands, I would propose, as the operations preferably to be executed, that serieses of triangles should be extended along different meridians, particularly those of the chief Observatories, and of some remarkable hills in the central parts of the island, till they fall into the sea on the south and north; and that, across these, a great number of serieses, in the parallels likewise of the chief Observatories, and of remarkable hills or eminencies, should be extended from the east to the west coast. The *Wrekin-hill*, on the eastern skirt of Shropshire, seems to be an eligible point, being in a central situation, sufficiently conspicuous, and yet not so high as to render the frequent access to it very inconvenient. If it should be found, that longitudes may be determined by the instrument to the degree of accuracy which is expected, and that it may likewise be advantageously applied, independently of the zenith sector, in tracing any parallel whose latitude has been already ascertained, or the parallel of any remarkable point, whose meridian is known, but not its latitude; then the number of these *parallel serieses* cannot be too much multiplied: for a great many of these operations executed with due care would
absolutely

absolutely give the figure of the earth, since the shortest distance from the surface to its axis would thereby be obtained.

The British dominions in the East-Indies offer a scene particularly favourable for the measurement of five degrees of latitude on the coast of Choromandel, as has been noticed by Mr. DALRYMPLE, F. R. S. in his Paper on the Marine Survey of that Coast. Two degrees of longitude, at each extremity of this arc, should likewise be measured.

The plains of Bengal, directly under the northern tropic, afford another situation where it would be of great consequence to determine the lengths of a degree or two of latitude, and as many of longitude. These two operations could not fail to be patronised by the East-India Company, who should defray the expence; since, whatever tended so much to the improvement of science in general, and so directly to that of navigation in particular, must be thought important to a Body of Merchants, whose power, as well as opulence, stand at this day unequalled in the mercantile history of the world.

But there is one operation yet to be mentioned that would contribute more than any other to the determination of the figure of the earth, which is, the accurate measurement of some degrees of longitude on the equator; because thereby the length of its semi-diameter would be immediately known, which hitherto has only been theoretically computed from the measured portions of the meridian. The Portuguese seem to be possessed of the most advantageous situation yet known on the globe for that purpose: for M. DE LA CONDAMINE has told us, that at the mouth of the river of Amazons, near the fort of *Macapa*, in three minutes north latitude, there are

extensive open plains where an operation of this sort would not have been so difficult as was at first imagined, when he proposed the scheme to the Academy of Sciences, a year before the voyage to *Quito* was thought of. Had this been adopted as the scene of their operations, instead of that elevated valley comprehended between the lofty ranges of the *Andes*, the business would have been much sooner accomplished, with infinitely less labour and fatigue, and still more satisfactorily, since degrees of longitude as well as of latitude would probably have been measured. Here then the Portuguese should determine the length of two or more degrees of longitude, and as many of latitude, by way of proof or correction of the Peruvian observations. A well conducted operation of this kind could not fail to add to the celebrity of a nation who, by their discovery of the new world in the west, and having opened the route by the Cape of Good Hope to the eastern side of the globe, may be considered as the founders of modern navigation.

With regard to operations in high northern latitudes, which would doubtless be of great importance, the Russian Empire must certainly afford a variety of situations, more or less difficult, for the purpose; and an Empress who commands so great a proportion of that region of the world, where it is said exploratory discoveries are at present carrying on by her order, can scarcely be supposed to suffer a reign, otherwise so brilliant, to expire, without directing something of this sort to be executed under the polar circle, or as near to the pole as the severity of the climate will permit, by way of confirmation or correction of the Lapland measurement, which yet stands single and by itself, without any collateral proof of its exactness. If such operations as those I have suggested towards the

close of this Paper were well executed in different parts of the world, little would then remain to be done towards the determination of the magnitude and figure of the earth, except multiplying, as much as possible, experiments with the same pendulums from the equator to very high southern and northern latitudes, that some judgement might thereby be formed of the similarity, or otherwise, of the two polar sides of the spheroid.

WILL. ROY.

Feb. 22, 1787.

E R R A T A.

In the Second Table subjoined to this Paper, page 228, under the head "Degrees of Longitude," in the column of the length of each degree in fathoms, l. 24. from the bottom (being that which answers to the Latitude of Greenwich) for 38161.69 read 38164.00; and in the next column of differences, l. 25. from the bottom, for 53.13 read 50.82; and the next line below, for 16.12 read 18.43.

In the Map of the Triangles, for SELSEY read SELSEY.

In the First Table, p. 227. the last line of the last column but one, for 38161.7 read 38164.0.

Comparison

Comparison of the observed length of the celestial arc of the meridian, comprehended

CELESTIAL ARC

Comprehended between the parallels of Greenwich and Perpignan.

Places of Observation.	Latitudes.	Differences of latitudes or celestial arcs.	Measured Arc.	Supposing the earth to be a sphere.			
		Deg.Min.&c.	Dec. parts	Fathoms.	Fathoms.	Error.	Fathoms.
	°	'	"				
1 { Greenwich Roy. Observ.	51	28	40 00	8 46 48 00	8.78	534343.	535927.
1 { Perpignan -	42	41	52 00				
Greenwich and M =	MP			diff. 0	26 50 52	0.4475	27350.
2 { Point M near Dunkirk	51	1	49 8	8 19 57 8	8.3325	506499.	507108. + 609
2 { Perpignan -	42	41	52 00				
Point M and Paris				diff. 2	11 35 8	2.1931	133417.
3 { Paris Royal Observatory	48	50	14 00	6 8 22 00	6.1394	373082.	373641. + 559
3 { Perpignan -	42	41	52 00				
Paris and Bourges				diff. 1	45 9 19	1.7525	106564.
4 { Bourges -	47	5	44 1	4 23 12 41	4.3869	266518.	266983. + 465
4 { Perpignan -	42	41	52 00				
Bourges and Rodés				diff. 2	43 51 5	2.6309	166009.
5 { Rodés -	44	21	13 36	1 39 21 36	1.656	100509.	100783. + 274
5 { Perpignan -	42	41	52 00				
Semi-diameter of the equator				-	-	3486975	3504545
Semi-axis between the poles				-	-	3486975	3484973
Ratio of the semi-diameters				-	-		179.047 to
Difference of the logarithms of the ratio				-	-		0.0024323
Some other chief properties.	Equator				-	-	60484.5
	Degrees of the meridian at the 45th				-	-	60777.6
	66° 20'				-	-	61194.3
	Pole				-	-	60859.1
	Excess of the degree of the meridian at the pole, above that at the equator				-	-	00000.0
Degrees of longitude at the	Equator				-	-	60859.1
	43° 32' south of France				-	-	44121.3
	51° 28' 40" Greenwich				-	-	37904.2

Gen. Roy's Account of a proposed trigonometrical Operation.

comprehended between the parallels of Greenwich and Perpignan, with the measured and computed

T E R R E S T R I A L A R C.

Computed arc on different *hypotheses*.

Supposing the Earth to be an oblate ellipsoid.

1		2		3		4		5	
ns.	Error.	Fathoms.	Error.	Fathoms.	Error.	Fathoms.	Error.	Fathoms.	Error.
7.		535588.		535088.		534970.		533942.	
0.		27331.		27301.		27294.		27241.	
7.	+ 2078	508257.	+ 1758	507787.	+ 1288	507676.	+ 1177	506701.	+ 202
4.	+ 517	134038.	+ 621	133747.	+ 330	133715.	+ 298	133454.	+ 37
3.	+ 1521	374374.	+ 1292	374040.	+ 958	373961.	+ 879	373247.	+ 155
7.	+ 443	106937.	+ 373	106834.	+ 270	106810.	+ 246	106605.	+ 41
6.	+ 1078	267437.	+ 918	267206.	+ 688	267151.	+ 633	266642.	+ 124
0.	+ 420	166518.	+ 509	166370.	+ 361	166334.	+ 325	166017.	+ 8
6.	+ 467	100919.	+ 410	100836.	+ 327	100817.	+ 308	100625.	+ 116
5	} 19572 178 047	3501802	} 18194 191.483	3497814	} 16190 215.06	3496934	} 15713 221.55	3489949	} 15174
3		3483608		3481624		3481221		3474775	
7 to 323		192.483 to 0.0022622		216.06 to 0.0020148		222.55 to 0.0019558		230. to 229 0.0018924	
	Error.		Error.		Error.		Error.		Error.
5	0.0	60484.5	0.0	60484.5	0.0	60484.5		60382.6	- 101.9
4	+ 215.8	60957.7	+ 180.1	60905.9	+ 128.3	60893.5	+ 115.9	60777.6	0.0
3	+ 148.0	61281.9	+ 87.6	61194.3	0.0	61173.4	- 20 9	61047.8	- 146.5
3		61437.1		61332.2		61308.4		61177.0	
8		952.6		847.7		823.9		794.4	
8		61117.9		61048.5		61033.0		60911.1	
2	+ 106.8	44430.4	+ 76.0	44355.7	+ 1.3	44341.7	- 12.7	44250.1	- 104.3
9		38181.3		38130.1		38117.7		37978.7	

computed lengths of the corresponding terrestrial arc between these parallels.

C.

					Supposing the earth to be a spheroid differing from the ellipsoid.			
					More prominent in middle latitudes than the ellipsoid.		Less prominent in middle latitudes than the ellipsoid of the same semi-diameters.	
					1		2	
rror.	Fathoms.	Error.	Fathoms.	Error.	Fathoms.	Error.	Fathoms.	Error.
	533860.		533751.		534997.		533737.	
	27230.		27206.		27296.		27243.	
202	506630.	+ 131	506545.	+ 46	507701.	+ 1202	506494.	- 5
37	133412.	- 5	133360.	- 57	133722.	+ 305	133438.	+ 21
155	373218.	+ 136	373185.	+ 103	373979.	+ 897	373056.	- 26
41	106582.	+ 18	106555.	- 9	106815.	+ 251	106570.	+ 6
124	266636.	+ 118	266630.	+ 112	267164.	+ 646	266486.	- 32
8	166005.	- 4	165991.	- 18	166372.	+ 333	165928.	- 81
116	100631.	+ 122	100639.	+ 130	100822.	+ 313	100558.	+ 49
5174	3487954 3476718 310.3 to 0.0014018	11236 309.3	3485542 3479087 540 to 539 0.0008050	6455	3496934 3481221 222.55 to 0.0019558	15713 221.55	3496740 3477210 179.4 to 0.0024274	19530 178.4
rror.		Error.		Error.		Error.		Error.
101.9	60484.5	0.0	60609.0	+ 124.5	60484.5	0.0	60484.5	0.0
0.0	60777.6	0.0	60777.6	0.0	60896.5	+ 118.9	60740.0	- 37.6
146.5	60977.7	- 216.6	60891.6	+ 302.7	61175.7	- 18.6	61203.7	+ 9.4
	61073.0		60947.0		61308.5		61506.6	
	588.5		338.0		824.0		1022.1	
104.3	60876.2 44213.4 37989.9	- 141.0	60834.1 44141.9 37932.4	- 212.5	61033.0 44341.8 38117.2	- 12.6	61029.6 44373.5 38161.7	+ 19.1

Table

Table of the degrees of the Earth constructed on the hypothesis of M. BOUGUER, where
squares

Latitudes.			Lengths of degrees of the meridian in fathoms.	Incre- ments above the deg. at the equator.	Diff. between deg. of the mer. and deg. of great circles perp. to the mer. or sum of the three equations.	Degrees of great circles perpendicu- lar to the meridian.	Degrees of great circles perpendic			
							Diff.	7 points = 78 ³ / ₄ .	Diff.	
	0	0	Diff.			Diff.				
	0	0	60484.50	0.06	545.12	61029.62	1.95	20.75	61008.87	59.08
	5	-	60484.56	0.06	547.01	61031.57	5.63	20.82	61010.75	59.29
	10	-	60485.43	0.93	551.77	61037.20	11.52	21.00	61016.20	59.80
	15	-	60489.08	4.58	559.64	61048.72	16.28	21.30	61027.42	60.66
	20	-	60498.49	13.99	566.51	61065.00	20.65	21.56	61043.44	61.40
	25	-	60517.11	31.27	568.54	61085.65	25.78	21.64	61064.01	61.62
	30	-	60548.38	63.88	563.15	61111.53	31.14	21.43	61090.10	61.04
	35	-	60595.13	110.63	547.54	61142.67	36.19	20.84	61121.83	59.35
	40	-	60658.99	174.49	519.87	61178.86	7.73	19.78	61159.08	56.35
			14.86							
	41	-	60673.85	189.35	512.74	61186.59	7.95	19.51	61167.08	55.58
	42	-	60689.40	204.90	505.14	61194.54	8.10	19.23	61175.31	54.75
	43	-	60705.62	221.12	497.02	61202.64	8.26	18.92	61183.72	53.87
	44	-	60722.50	238.00	488.40	61210.90	8.40	18.59	61192.31	52.93
	45	-	60740.02	255.52	479.28	61219.30	8.51	18.24	61201.06	51.95
	46	-	60758.17	273.67	469.64	61227.81	8.66	17.86	61209.95	50.90
	47	-	60776.92	292.42	459.55	61236.47	8.77	17.49	61218.98	49.81
	48	-	60796.24	311.74	449.00	61245.24	7.36	17.09	61228.15	48.66
Paris	48	50 14	60812.83	337	439.77	61252.60	1.43	16.74	61235.86	47.66
	49	-	60816.10	331.60	437.93	61254.03	8.96	16.67	61237.36	47.46
	50	-	60836.47	351.97	426.52	61262.99	0.76	16.23	61246.76	46.23
			1.72							
	50	5	60838.19	353.69	425.56	61263.75	0.74	16.20	61247.55	46.12
	50	10	60839.92	355.42	424.57	61264.49	0.74	16.16	61248.33	46.02
	50	15	60841.64	357.14	423.59	61265.23	0.78	16.12	61249.11	45.91
	50	20	60843.37	358.87	422.64	61266.01	0.76	16.09	61249.92	45.80
	50	25	60845.11	360.61	421.66	61266.77	0.74	16.05	61250.72	45.70
	50	30	60846.84	362.34	420.67	61267.51	0.75	16.01	61251.50	45.60
	50	35	60848.58	364.08	419.68	61268.26	0.74	15.97	61252.29	45.49
	50	40	60850.32	365.82	418.68	61269.00	0.79	15.93	61253.07	45.38
	50	45	60852.07	367.57	417.72	61269.79	0.74	15.90	61253.89	45.27
	50	50	60853.82	369.32	416.71	61270.53	0.74	15.86	61254.67	45.17
	50	55	60855.57	371.07	415.70	61271.27	0.74	15.82	61255.45	45.06
	51	0	60857.32	372.82	414.70	61272.02	0.75	15.78	61256.24	44.95
			1.76							
	51	5	60859.08	374.58	413.68	61272.76	0.77	15.74	61257.02	44.84
	51	10	60860.84	376.34	412.69	61273.53	0.78	15.71	61257.82	44.73
	51	15	60862.61	378.11	411.70	61274.31	0.74	15.67	61258.64	44.62
	51	20	60864.37	379.87	410.68	61275.05	0.75	15.63	61259.42	44.51
	51	25	60866.14	381.64	409.66	61275.80	0.54	15.59	61260.21	44.40
Greenwich	51	28 40	60867.44	382.94	408.90	61276.34	0.21	15.56	61260.78	44.32
	51	30	60867.92	383.42	408.63	61276.55	0.75	15.55	61261.00	44.29
	51	35	60869.69	385.19	407.61	61277.30	0.74	15.51	61261.79	44.18
	51	40	60871.47	386.97	406.57	61278.04	0.80	15.47	61262.57	44.07
	51	45	60873.25	388.75	405.59	61278.84	0.74	15.44	61263.40	43.95
	51	50	60875.04	390.54	404.54	61279.58	0.75	15.40	61264.18	43.84
	51	55	60876.82	392.32	403.51	61280.33	0.76	15.36	61264.97	43.73
	52	0	60878.61	394.11	402.48	61281.09	9.09	15.32	61265.77	43.62
			21.69							
	53	-	60900.30	415.80	389.88	61290.18	9.13	14.84	61275.34	42.26
	54	-	60922.35	437.85	376.96	61299.31	9.14	14.35	61284.96	40.86
	55	-	60944.70	460.20	363.75	61308.45	9.14	13.84	61294.61	39.43
	56	-	60967.33	482.83	350.26	61317.59	9.10	13.33	61304.26	37.96
			22.83					12.81	61313.88	36.41

where the increments to the degrees of the meridian above that at the equator follow the ratio of squares of the fines of the latitudes.

at circles oblique to the meridian, the differences between the meridional degrees, and those of great circles perpendicular to them, being in the ratio of the squares of the fines of obliquity.

Diff.	6 points = $67^{\circ}\frac{1}{2}$.	Diff.	5 points = $56^{\circ}\frac{1}{4}$.	Diff.	4 points = 45° .	Diff.	3 points = $33^{\circ}\frac{1}{2}$.	Diff.	2 points = $22^{\circ}\frac{1}{2}$.	Diff.	1 point = $11^{\circ}\frac{1}{4}$.	Degree
59.68	60949.79	88.43	60861.36	104.20	60757.16	104.40	60652.76	88.43	60564.33	59.08	60505.25	61029.
59.29	60951.46	88.73	60862.73	104.66	60758.07	104.67	60653.40	88.73	60564.67	59.22	60505.45	60799.
59.80	60956.40	89.51	60866.89	105.57	60761.32	105.58	60655.74	89.51	60566.23	59.80	60506.43	60109.
60.66	60966.76	90.87	60875.89	107.63	60768.26	106.44	60661.82	90.78	60571.04	60.66	60510.38	58968.
61.40	60982.04	91.90	60890.14	108.39	60781.75	108.40	60673.35	91.90	60581.45	61.40	60520.05	57382.
61.62	61002.39	92.24	60910.15	108.77	60801.38	108.79	60692.59	92.22	60600.37	61.62	60538.75	55362.
61.04	61029.06	91.35	60937.71	107.75	60829.96	107.76	60722.20	91.35	60630.85	61.04	60569.81	52924.
59.35	61062.48	88.81	60973.67	104.77	60868.90	104.77	60764.13	88.81	60675.32	59.35	60615.97	50085.
56.35	61102.73	84.33	61018.40	99.48	60918.92	99.47	60819.45	84.33	60735.12	56.34	60678.78	46865.
55.58	61111.50	83.17	61028.33	98.11	60930.22	98.11	60832.11	83.17	60748.94	55.58	60693.36	46178.
54.75	61120.56	81.94	61038.62	96.65	60941.97	96.65	60845.32	81.94	60763.38	54.75	60708.63	45476.
53.87	61129.85	80.61	61049.24	95.11	60954.13	95.10	60859.03	80.62	60778.41	53.87	60724.54	44760.
52.93	61139.38	79.23	61060.15	93.45	60966.70	93.45	60873.25	79.23	60794.02	52.93	60741.09	44031.
51.95	61149.11	77.74	61071.37	91.71	60979.66	91.71	60887.95	77.74	60810.21	51.95	60758.26	43288.
50.90	61159.05	76.18	61082.87	89.87	60993.00	89.86	60903.14	76.19	60826.95	50.91	60776.04	42532.
49.81	61169.17	74.55	61094.62	87.93	61006.69	87.93	60918.76	74.54	60844.22	49.81	60794.41	41763.
48.66	61179.49	72.84	61106.65	85.91	61020.74	85.91	60934.83	72.84	60861.99	48.66	60813.33	40981.
47.66	61188.20	71.34	61116.86	84.15	61032.71	84.14	60948.57	71.34	60877.23	47.66	60829.57	40316.
47.46	61189.90	71.04	61118.86	83.80	61035.06	83.79	60951.27	71.04	60880.23	47.46	60832.77	40186.
46.23	61200.53	69.19	61131.34	81.61	61049.73	81.61	60968.12	69.19	60898.93	46.23	60852.70	39379.
46.12	61201.43	69.03	61132.40	81.43	61050.97	81.43	60969.54	69.03	60900.51	46.12	60854.39	39311.
46.02	61202.31	68.87	61133.44	81.23	61052.21	81.24	60970.97	68.87	60902.10	46.02	60856.08	39243.
45.91	61203.20	68.71	61134.49	81.06	61053.43	81.05	60972.38	68.71	60903.67	45.91	60857.76	39175.
45.80	61204.12	68.56	61135.56	80.87	61054.69	80.87	60973.82	68.56	60905.26	45.80	60859.46	39107.
45.70	61205.02	68.40	61136.62	80.68	61055.94	80.68	60975.26	68.40	60906.86	45.70	60861.16	39039.
45.60	61205.90	68.23	61137.67	80.50	61057.17	80.49	60976.68	68.23	60908.45	45.60	60862.85	38970.
45.49	61206.80	68.08	61138.72	80.30	61058.42	80.30	60978.12	68.08	60910.04	45.49	60864.55	38902.
45.38	61207.69	67.92	61139.77	80.11	61059.66	80.11	60979.55	67.92	60911.63	45.37	60866.26	38834.
45.27	61208.62	67.76	61140.86	79.93	61060.93	79.93	60981.00	67.76	60913.24	45.27	60867.97	38765.
45.17	61209.50	67.59	61141.91	79.74	61062.17	79.73	60982.44	67.59	60914.85	45.17	60869.68	38697.
45.06	61210.39	67.43	61142.96	79.54	61063.42	79.54	60983.88	67.43	60916.45	45.06	60871.39	38628.
44.95	61211.29	67.27	61144.02	79.35	61064.67	79.35	60985.32	67.27	60918.05	44.95	60873.10	38559.
44.84	61212.18	67.11	61145.07	79.15	61065.92	79.15	60986.77	67.11	60919.66	44.84	60874.82	38490.
44.73	61213.09	66.94	61146.15	78.96	61067.19	78.97	60988.22	66.94	60921.28	44.73	60876.55	38422.
44.62	61214.02	66.79	61147.23	78.77	61068.46	78.78	60989.68	66.78	60922.90	44.62	60878.28	38353.
44.51	61214.91	66.62	61148.29	78.58	61069.71	78.58	60991.13	66.62	60924.51	44.51	60880.00	38283.
44.40	61215.81	66.45	61149.36	78.39	61070.97	78.39	60992.58	66.45	60926.13	44.40	60881.73	38214.
44.32	61216.46	66.33	61150.13	78.24	61071.89	78.24	60993.65	66.33	60927.32	44.32	60883.00	38161.
44.29	61216.71	66.29	61150.42	78.18	61072.24	78.19	60994.05	66.29	60927.76	44.29	60883.47	38145.
44.18	61217.61	66.12	61151.49	78.00	61073.49	77.99	60995.50	66.12	60929.38	44.18	60885.20	38076.
44.07	61218.50	65.95	61152.55	77.79	61074.76	77.80	60996.96	65.95	60931.01	44.07	60886.94	38006.
43.95	61219.45	65.80	61153.65	77.60	61076.05	77.61	60998.44	65.80	60932.64	43.95	60888.69	37937.
43.84	61220.34	65.62	61154.72	77.41	61077.31	77.41	60999.90	65.62	60934.28	43.84	60890.44	37867.
43.73	61221.24	65.46	61155.78	77.21	61078.57	77.20	61001.37	65.46	60935.91	43.73	60892.18	37798.
43.62	61222.15	65.29	61156.86	77.01	61079.85	77.01	61002.84	65.29	60937.55	43.62	60893.93	37728.
42.26	61233.08	63.24	61169.84	74.60	61095.24	74.60	61020.64	63.24	60957.40	42.26	60915.14	36885.
40.86	61244.10	61.14	61182.96	72.13	61110.83	72.13	61038.70	61.15	60977.55	40.85	60936.70	36030.
39.43	61255.18	59.00	61196.18	69.60	61126.58	69.61	61056.97	59.00	60997.97	39.43	60958.54	35165.
37.96	61266.30	56.82	61209.48	67.02	61142.46	67.02	61075.44	56.82	61018.62	37.96	60980.66	34288.

ratio of the 4th power, or squared

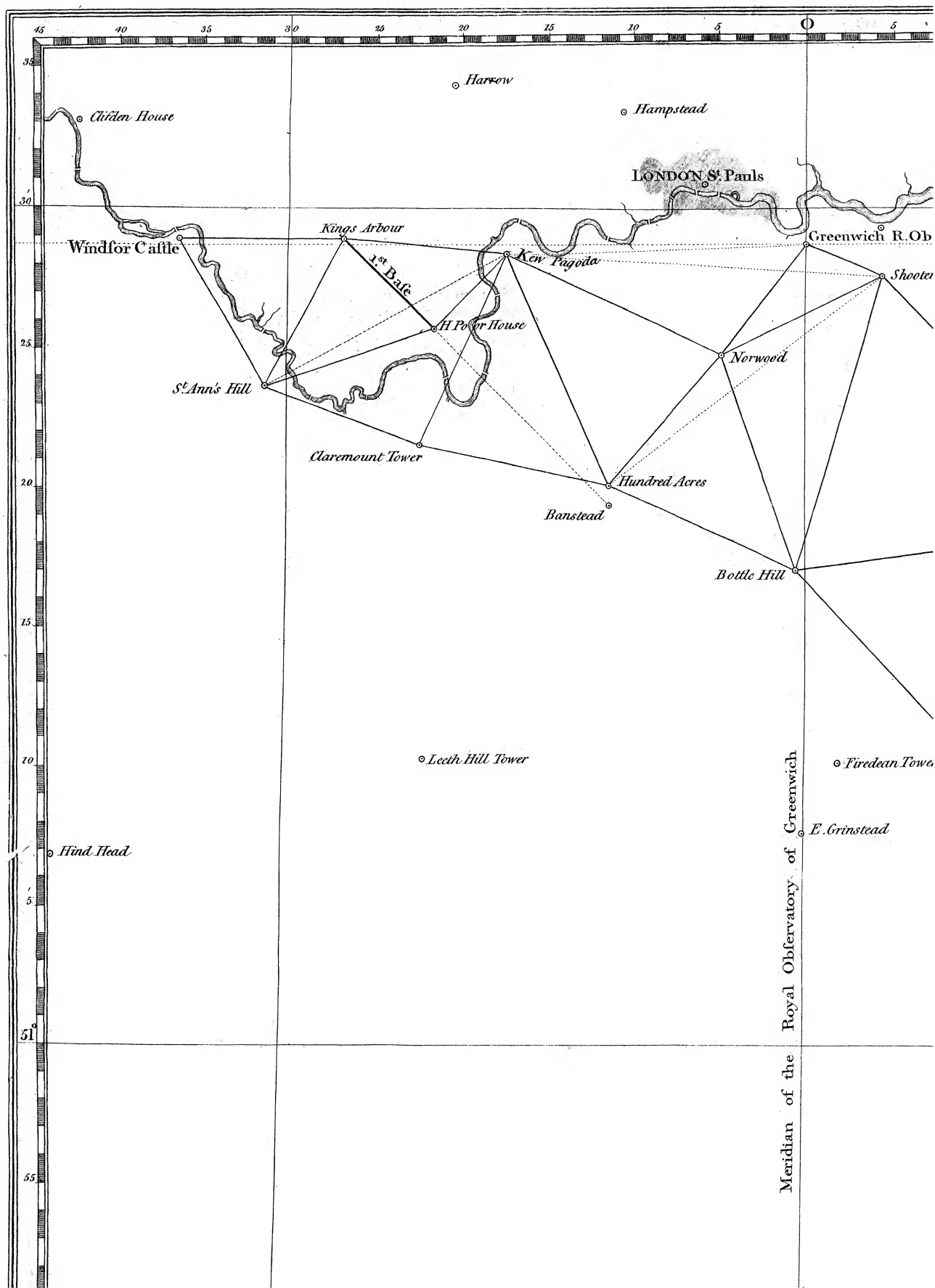
Degrees of longitude.		Latitudes.		
	Diff.			
61029.62		0	0	0
60799.34	230.28	5	-	-
60109.86	689.48	10	-	-
58968.53	1141.33	15	-	-
57382.33	1586.20	20	-	-
55362.39	2019.94	25	-	-
52924.13	2438.26	30	-	-
50085.13	2839.00	35	-	-
46865.74	3219.39	40	-	-
	687.62			
46178.12		41	-	-
45476.40	701.72	42	-	-
44760.77	715.63	43	-	-
44031.43	729.34	44	-	-
43288.58	742.85	45	-	-
42532.41	756.17	46	-	-
41763.17	769.24	47	-	-
40981.06	782.11	48	-	-
40316.51	664.55	48	50	14
40186.26	130.25	49	-	-
39379.09	807.17	50	-	-
	67.80			
39311.29	67.90	50	5	-
39243.39	68.02	50	10	-
39175.37	68.05	50	15	-
39107.32	68.14	50	20	-
39039.18	68.24	50	25	-
38970.94	68.32	50	30	-
38902.62	68.40	50	35	-
38834.22	68.50	50	40	-
38765.72	68.57	50	45	-
38697.15	68.67	50	50	-
38628.48	68.76	50	55	-
38559.72		51	0	0
	68.82			
38490.90	68.90	51	5	-
38422.00	68.97	51	10	-
38353.03	69.06	51	15	-
38283.97	69.15	51	20	-
38214.82	69.15	51	25	-
38161.69	53.13	51	28	40
38145.57	10.12	51	30	-
38076.24	69.33	51	35	-
38006.83	69.41	51	40	-
37937.34	69.49	51	45	-
37867.77	69.57	51	50	-
37798.12	69.65	51	55	-
37728.40	69.72	52	0	0
	843.05			
36885.35		53	-	-
36030.83	854.52	54	-	-
35165.08	865.75	55	-	-
34288.36	876.72	56	-	-

Equal to deg. of long. on the equator.	54	-	-	60922.35	22.35	437.85	376.96	61299.31	9.14	14.35	61284.90	40.86
	55	-	-	60944.70	22.63	460.20	363.75	61308.45	9.14	13.84	61294.61	39.43
	56	-	-	60967.33	22.83	482.83	350.26	61317.59	9.10	13.33	61304.26	37.96
	57	-	-	60990.16	23.00	505.66	336.53	61326.69	9.07	12.81	61313.88	36.47
	58	-	-	61013.16	16.46	528.66	322.60	61335.76	6.45	12.28	61323.48	34.96
	58	42	47.4	61029.62	6.65	545.12	312.59	61342.21	2.57	11.90	61330.31	33.88
	59	-	0	61036.27	23.16	551.77	308.51	61344.78	8.94	11.74	61333.04	33.44
	60	-	0	61059.43	114.67	574.93	294.29	61353.72	42.98	11.20	61342.52	31.90
Deg. meaf. in Lapland.	65	-	-	61174.10	29.65	689.60	222.60	61396.70	10.80	8.47	61388.23	24.13
	66	20	-	61203.75	77.71	719.25	203.75	61407.50	27.89	7.75	61399.75	22.09
	70	-	-	61281.46	92.79	796.96	153.70	61435.16	31.80	5.85	61429.31	16.66
	75	-	-	61374.25	71.64	889.75	92.71	61466.96	24.31	3.53	61463.43	10.05
	80	-	-	61445.89	45.24	961.39	45.38	61491.27	12.33	1.73	61489.54	4.92
	85	-	-	61491.13	15.47	1006.63	12.47	61503.60	3.00	0.47	61503.13	1.36
	90	-	-	61506.60		1022.10	0.00	61506.60		0.00	61506.60	0.00

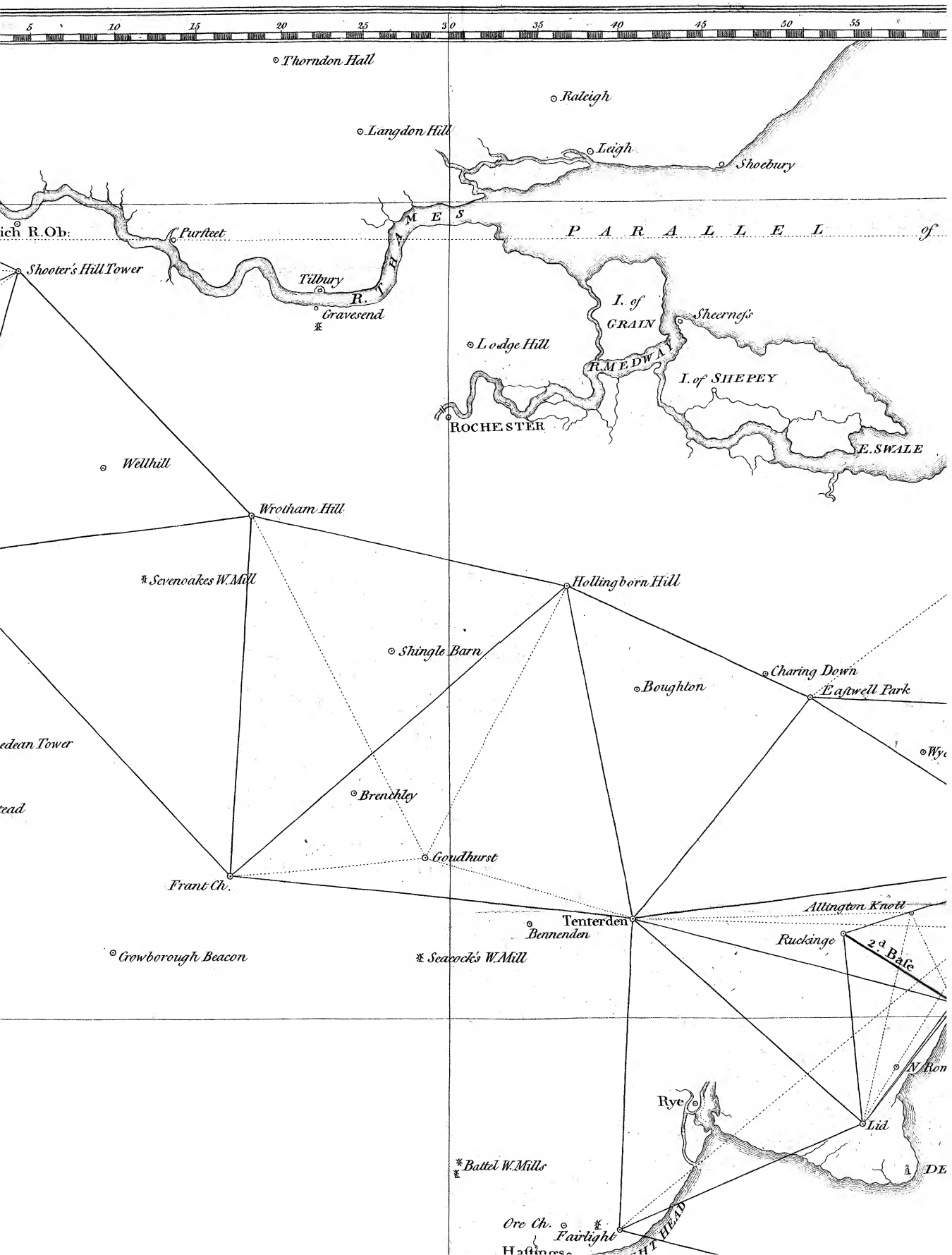
[illegible]

36030.83	854.52	Equal to deg. of long. on the equator. }	53	-	-
35165.08	865.75		54	-	-
34288.36	876.72		55	-	-
33400.91	887.45		56	-	-
32503.54	897.37		57	-	-
	647.12		58	-	-
31856.42	261.52		58	42	47.4
31594.90	918.04	Deg. meas. in Lapland. }	59	-	-
30676.86			60	-	-
	4729.49				
25947.37	1297.48		65	-	-
24649.89	3637.83		66	20	-
21012.06	5103.24		70	-	-
15908.82	5330.97		75	-	-
10677.85	5317.45		80	-	-
5360.40	5360 40		85	-	-
00.00			90	-	-

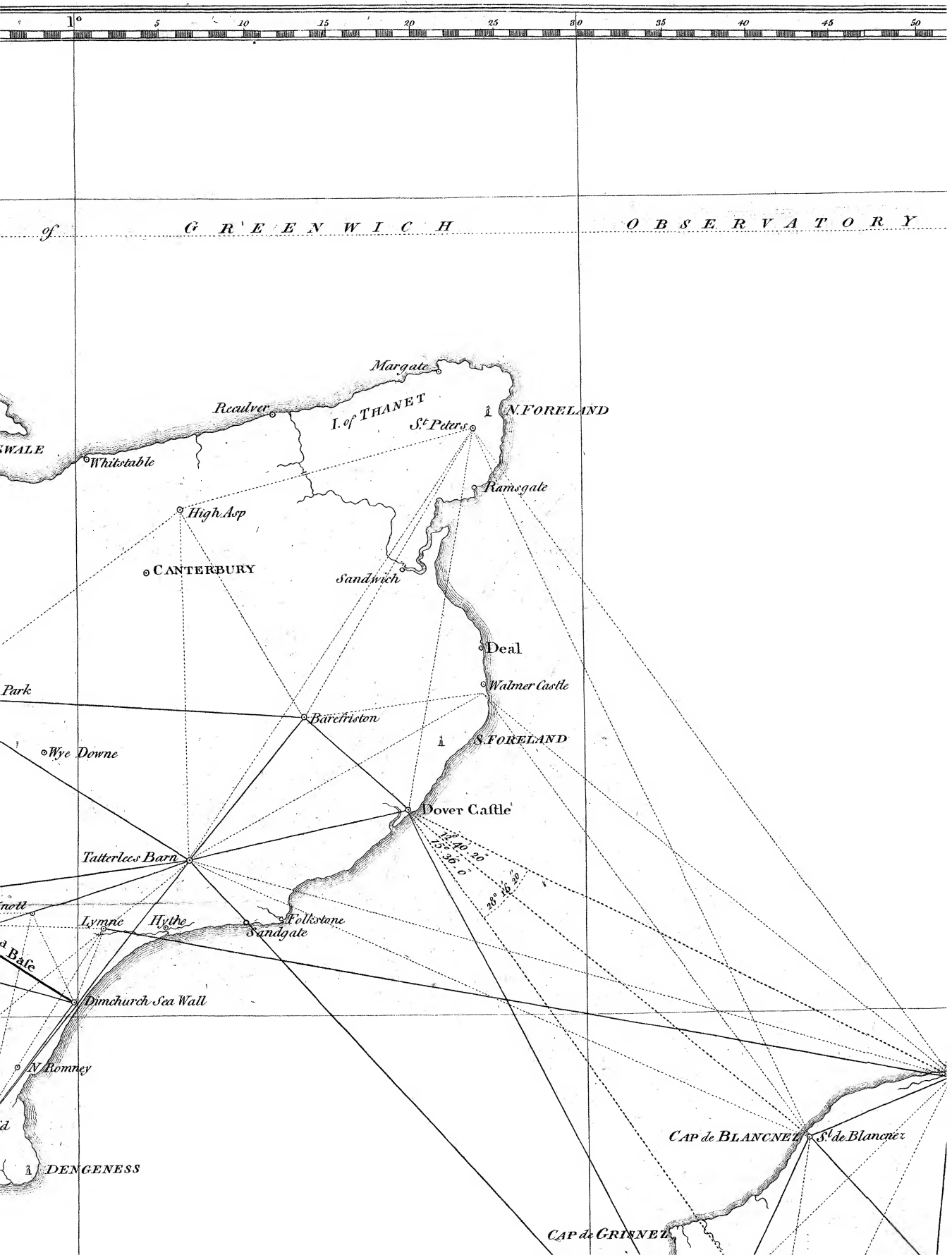
MAP *shewing nearly the DISPOSITION*



ION of the TRIANGLES, whereby it is proposed to c

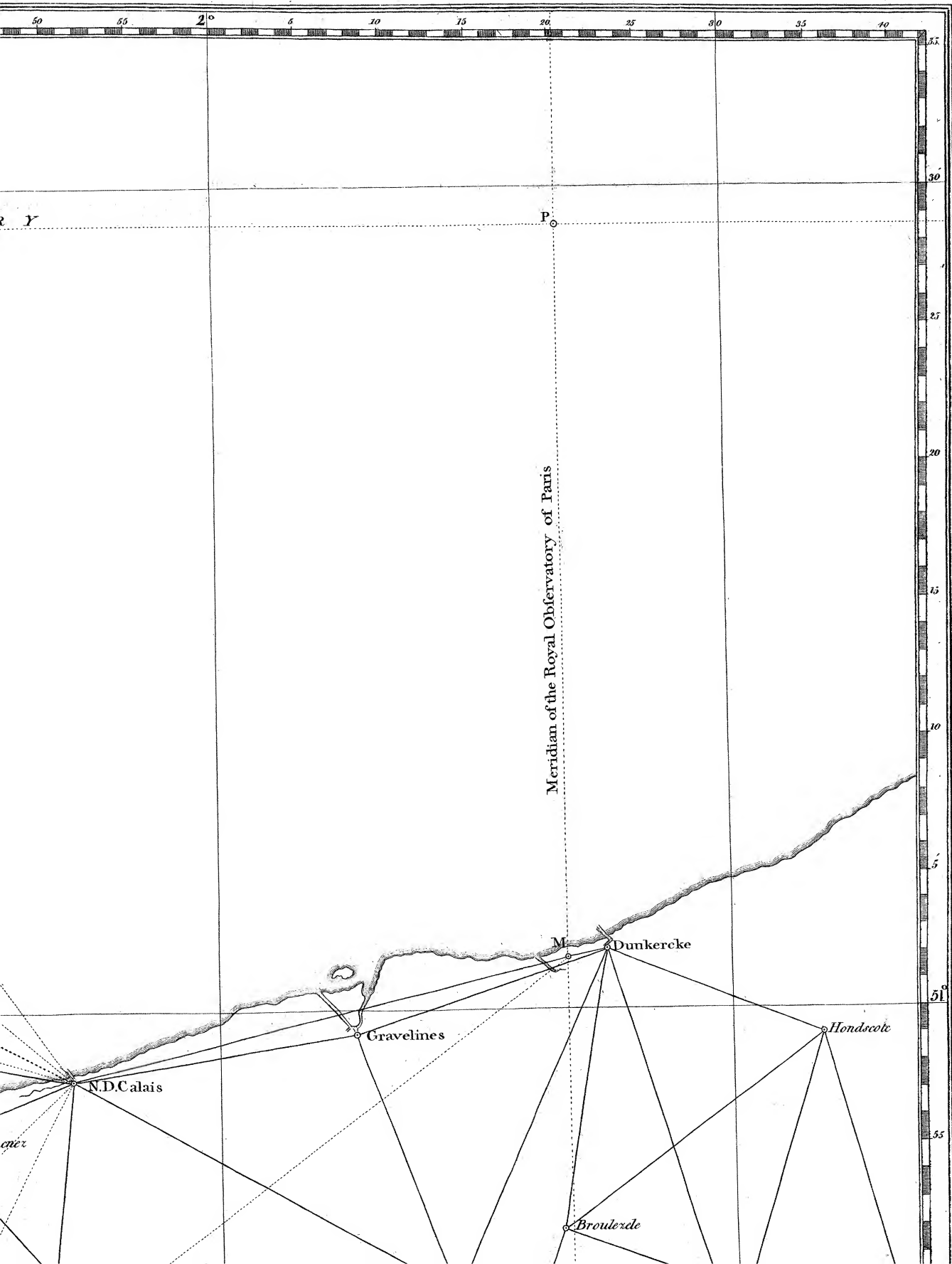


to determine the Relative Situations of the Royal Obs

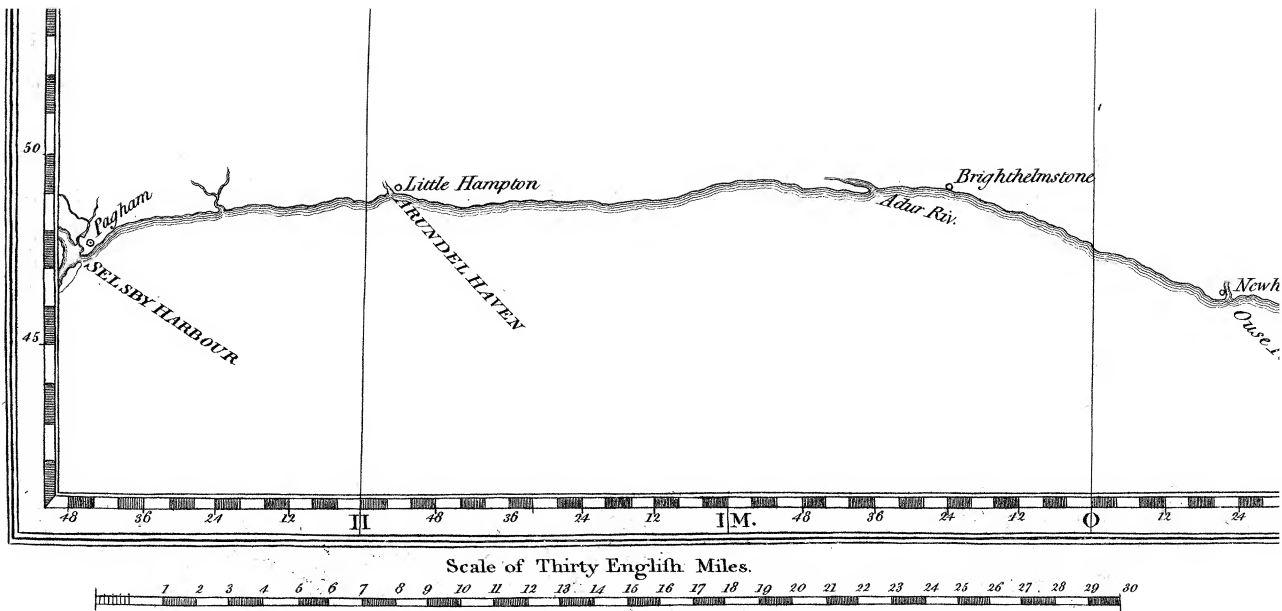


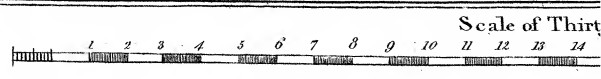
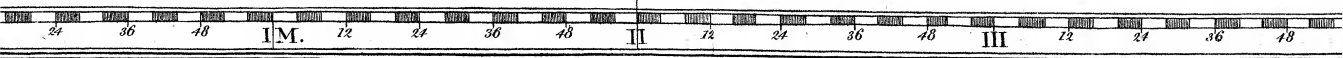
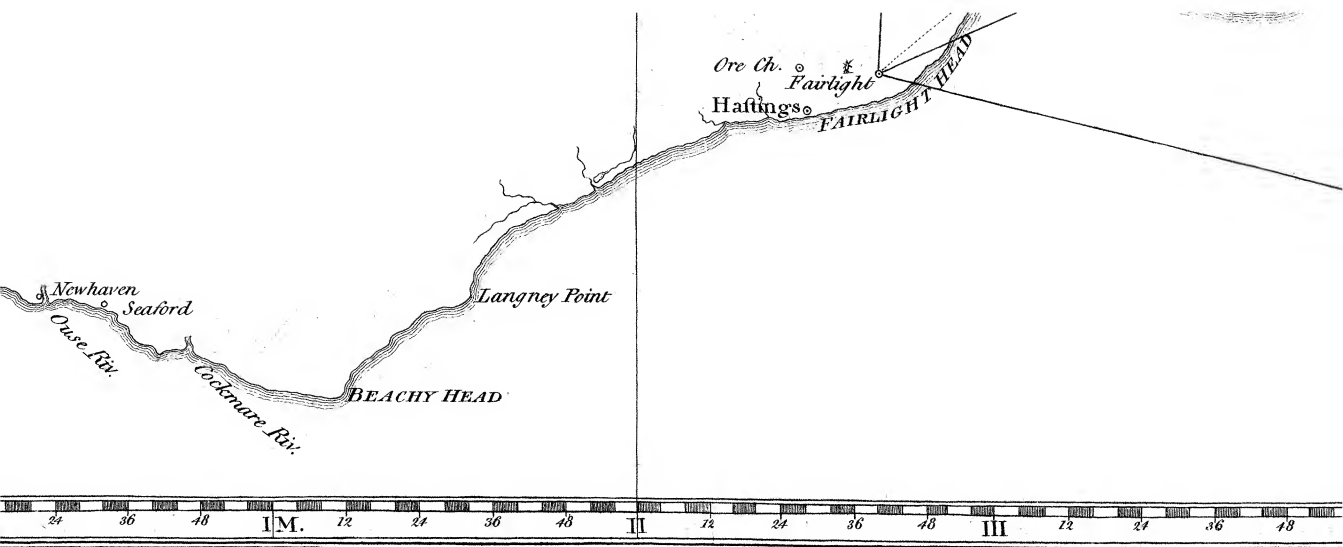
Observatories of GREENWICH and PARIS.

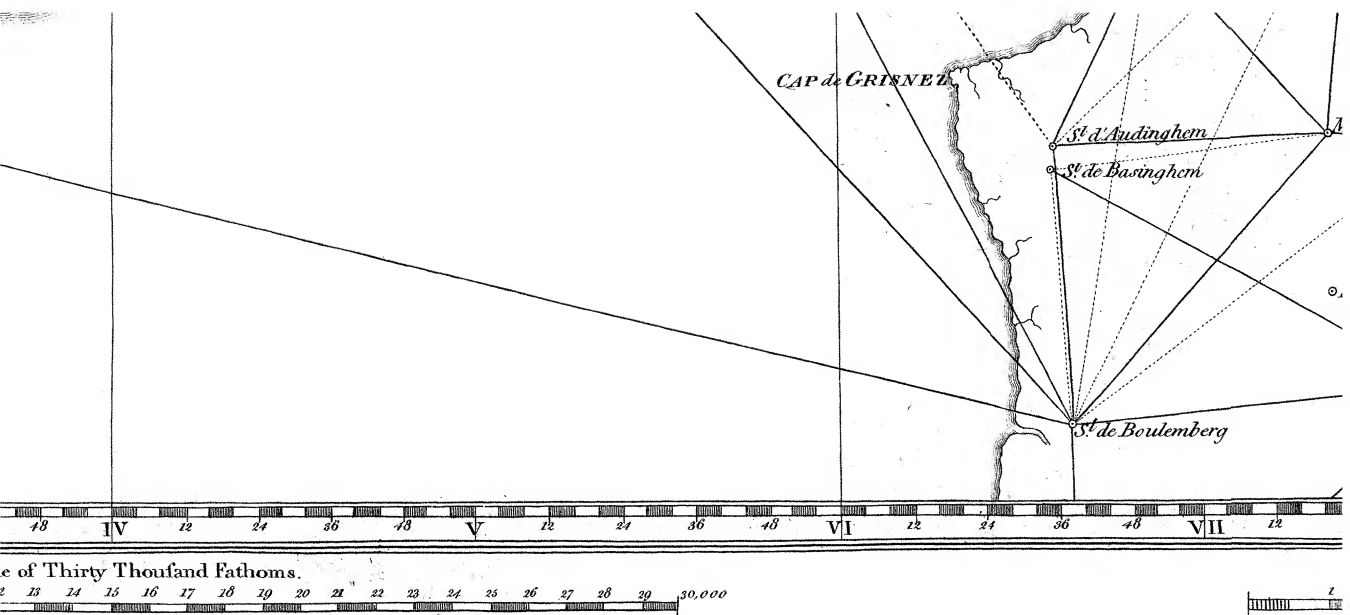
Philos. Trans. Vol. LXXVII. Tab. IX. p. 228.







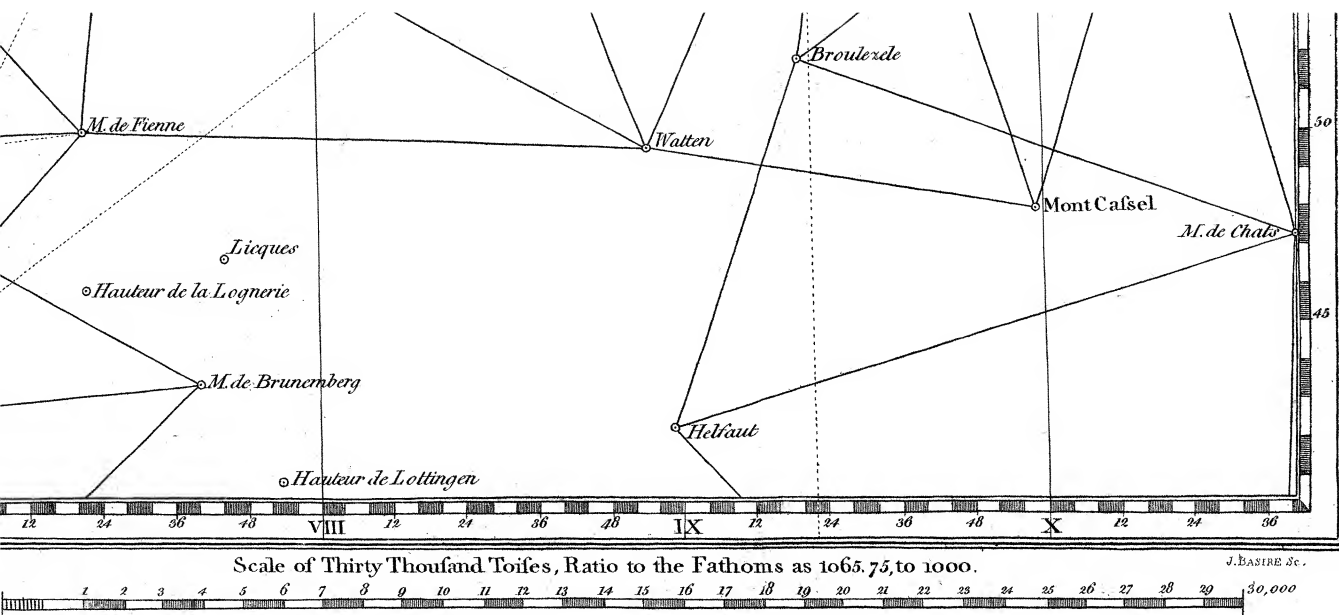


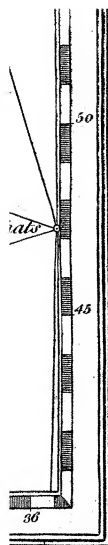


Scale of Thirty Thousand Fathoms.

2 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30,000

1





ASTIRE 56.

30,000

A P P E N D I X.

A Supplement to Major-General Roy's Account of the Mode proposed to be followed in determining the relative Situation of the Royal Observatories of Greenwich and Paris. See p. 188.

Read Nov. 8, 1787.

IN the account of the proposed trigonometrical operation for determining the difference between the meridians of the Royal Observatories of Greenwich and Paris, I have, at p. 221. and 222. had occasion to remark on an inconsistency found in the sum of the three equations, as stated by M. BOUGUER, for obtaining the lengths of degrees of great circles perpendicular to the meridian, above their corresponding degrees of latitude, without having been aware of the true source of the error, for which I am indebted to the investigations of the Astronomer Royal; who, having found it out, obligingly communicated the same to me, about the time of the annual recess of the Society in the end of June last.

At p. 289. and again at p. 313. and 314. of M. BOUGUER's Book, the subtractive branch of the equation, or the part of DG the gravicentric arc, answering to the difference between the *radii* of curvature at the equator and given latitude, has erroneously been expressed in words $\frac{1}{4}$ ths instead of $\frac{1}{5}$ ths, which the algebraic *formula* justly gives for it, and according to which M. BOUGUER's table has been accurately computed. Not suspecting any thing of this sort, no intima-

tion of it appearing among the *errata*, and no notice having been hitherto taken, as far as I know, of such a mistake existing in that justly celebrated work, now so many years in circulation throughout the world; instead therefore of $\frac{4}{15}$ ths, or $\frac{8}{30}$ parts of the third proportional, the last additive member of the equation, I substituted another (with a certain modification, however, as stated in the before-mentioned pages) amounting only to $\frac{8}{370}$ parts of the third proportional, being the highest that would apply to the whole quadrant, without producing absurd results. Thus I obtained approximate degrees of great circles and of longitude, differing but little from those of M. BOUGUER, and compensating in a great measure, although not altogether, for the then undiscovered cause of the mistake of $\frac{1}{20}$ th part of the arc DG; for $\frac{4}{5} - \frac{3}{4} = \frac{1}{20}$.

In this state of the case, I have judged it incumbent on me to annex a supplementary table, where the degrees of great circles and of longitude are accurately computed by the corrected subtractive branch $\frac{4}{5}$ ths of DG instead of $\frac{3}{4}$ ths, as it now stands in the original table. From inspection it will appear, that the maximum of correction amounts nearly to $5\frac{1}{2}$ fathoms at the 70th degree of latitude, diminishing gradually from thence to the pole on one side, and the equator on the other, where it vanishes. The maximum of correction for the degrees of longitude, amounting to about $2\frac{1}{2}$ fathoms, is applicable between the 58th and 59th degree of latitude, where M. BOUGUER's degree of the meridian becomes equal to his degree of longitude on the equator. From this point, it diminishes gradually to the pole on one side, and the equator on the other, where it in like manner disappears.

With regard to degrees of great circles situated obliquely to the meridian, it is sufficiently obvious, that they are so little affected as to render it of but small importance whether they

are corrected or not ; but for such as are scrupulous to fractional parts of fathoms, these may satisfy themselves with great facility and exactness, by attending to and proportioning by the common tabular differences, as in the following example of the application of the correction to oblique degrees, in the latitude of Greenwich $51^{\circ} 28' 40''$.

	Obliquity.	Original tabular difference.	Supp. correc- tion.	Corrected difference.	Corrected degree.
	$\begin{smallmatrix} 0 & 0 \\ 45 & 0 \\ 33 & 45 \\ 56 & 15 \\ 22 & 30 \\ 67 & 30 \\ 11 & 15 \\ 78 & 45 \\ 0 & 0 \\ 90 & 0 \end{smallmatrix}$	$\begin{smallmatrix} \text{Fath.} \\ 204.45 \\ 78.24 \\ 66.33 \\ 44.32 \\ 15.56 \end{smallmatrix}$	$\begin{smallmatrix} \text{Fath.} \\ -1.65 \\ -0.63 \\ -0.54 \\ -0.36 \\ -0.12 \end{smallmatrix}$	$\begin{smallmatrix} \text{Fath.} \\ 202.80 \\ 77.61 \\ 65.79 \\ 43.96 \\ 15.44 \end{smallmatrix}$	$\begin{smallmatrix} \text{Fath.} \\ 61070.20 \\ 60992.63 \\ 61147.85 \\ 60926.84 \\ 61213.64 \\ 60882.88 \\ 61257.60 \\ 60867.44 \\ 61273.04 \end{smallmatrix}$
Application of the correction to the degrees of great circles, situated obliquely to the meridian, in the latitude of Greenwich.					

Here it is to be observed, that half of the correction is constantly to be applied at the 45th degree of obliquity ; and since the differences between the terms in the progression equally removed on either side from 45° , are always equal to each other, it follows, that the corrected differences are to be applied with the contrary sign, those between 45° and the meridian being in diminution, and these between 45° and the east or west points being in augmentation of the length, till, in the first case, the degree becomes equal to that of latitude ; and in the last to that of a great circle perpendicular to the meridian.

The Table of Comparison, p. 227. is no where affected by the alteration which has been the subject of this discussion, except in the two last lines from the bottom, as adverted to among the *errata* subjoined to this supplement.

This intimation of M. BOUGUER's mistake in expressing his formula in words, was accompanied with the conversion of
that

that formula by Dr. MASKELYNE into the following, adapted to natural sines, $\frac{a \times 11 + 8 \times \cos. 2 \text{ lat} - 3 \cos. 4 \text{ lat.}}{30}$. a is the excess of the radius of curvature at the pole above that at the equator, and the sign of either term is only to be changed when the doubled or quadrupled latitude become greater than 90° , and less than 270° .

I embrace this opportunity of mentioning another circumstance, wholly unknown to me at the time my Paper was composed. From what has been there said, at p. 216. and so on to p. 220. it will probably be inferred, that I considered the proposed mode of determining the differences of longitude by the observations of the pole star, made with a very accurate instrument, rather as new, not knowing that the same idea, or one nearly the same, had before occurred to the Rev. Mr. MICHELL, and been treated on by him in his very ingenious Paper in the Philosophical Transactions, Vol. LVI. for the year 1766. That I must have read that valuable performance about the time of its publication is not to be doubted; but in the lapse of so many years, every trace of it had gone from my remembrance, otherwise I would have most certainly referred to it in the proper place, and with the attention that it so well deserves. However, without entering here into particulars, it will obviously appear, that the one has not been borrowed from the other.

E R R A T A.

In p. 195. l. 27. for between Gravelines and Calais read between Watten and Gravelines. And in l. 28. for $56^\circ 42' 0''$ read $46^\circ 52' 0''$.

In p. 197. in the distance of the parallel of Rodés from that of Dunkirk, for $6^\circ 50' 51'' 14'''$ read $6^\circ 40' 51'' 14'''$.

In the last line of the note at the bottom of p. 217. instead of $1^\circ 49' 8''.8$, read $1^\circ 49' 48''.8$.

In p. 221. line 6. from the top, instead of $\frac{7}{15}$ ths, read $\frac{8}{15}$ ths.

In the Table of Comparison, p. 227. the last line but one of the last column but one, for 44373.5 read 44372.0. In the next line below, for 38164.0, as formerly corrected, read 38162.0. And in the right-hand column, last line but one from the bottom, for +19.1, read +17.6.

Supplementary Table for the Degree

Places.	Latitudes.	Sum of the 3 equat. ths of DG being subtrac- tive.	Supple- mentary correction for deg. of great circles.	Degrees of great circles perpendicular to the meridian cor- rected.	Supple- mentary correction for deg. of long.	Degrees of longitude corrected.	Places.
Equator	° ' "			Diff.		Diff.	
	0 0 0	545.12	±0.00	61029.62	0.00	61029.62	
	5 - -	547.081	+0.07	61031.64	+0.04	60799.38	230.24
	10 - -	552.595	+0.82	61038.02	+0.86	60110.72	688.66
	15 - -	558.739	-0.90	61047.82	-0.85	58967.68	1143.04
	20 - -	565.811	-0.70	61064.30	-0.67	57381.66	1586.02
	25 - -	567.715	-0.83	61084.82	-0.74	55361.65	2020.01
	30 - -	562.156	-0.99	61110.54	-0.86	52923.27	2438.38
	35 - -	546.285	-1.25	61141.42	-1.02	50084.11	2839.16
	40 - -	518.144	-1.73	61177.13	-1.33	46864.41	3219.70
				7.68			687.63
	41 - -	510.955	-1.78	61184.81	-1.34	46176.78	701.78
	42 - -	503.235	-1.90	61192.64	-1.40	45475.00	715.74
	43 - -	494.946	-2.07	61200.57	-1.51	44759.26	738.29
S. of France	43 32	490.392	-2.11	61204.93	-1.53	44371.97	342.08
	44 - -	486.244	-2.16	61208.74	-1.54	44029.89	742.97
	45 - -	476.984	-2.30	61217.00	-1.66	43286.92	756.18
	46 - -	467.216	-2.42	61225.39	-1.67	42530.74	769.34
	47 - -	456.972	-2.58	61233.89	-1.77	41761.40	782.27
	48 - -	446.252	-2.75	61242.49	-1.83	40979.23	664.59
Paris	48 50 14	436.936	-2.83	61249.77	-1.87	40314.64	130.25
	49 - -	435.084	-2.85	61251.18	-1.87	40184.39	807.24
	50 - -	423.488	-3.03	61259.96	-1.94	39377.15	67.83
				0.73			67.90
	50 5 -	422.504	-3.06	61260.69	-1.97	39309.32	68.00
	50 10 -	421.508	-3.06	61261.43	-1.97	39241.42	68.09
	50 15 -	420.523	-3.07	61262.16	-1.95	39173.42	68.14
	50 20 -	419.534	-3.11	61262.90	-1.99	39105.33	68.26
	50 25 -	418.526	-3.13	61263.64	-1.99	39037.19	68.33
	50 30 -	417.531	-3.14	61264.37	-2.01	38968.93	68.41
	50 35 -	416.528	-3.15	61265.11	-2.02	38900.60	68.50
	50 40 -	415.524	-3.16	61265.84	-2.03	38832.19	68.58
	50 45 -	414.515	-3.20	61266.59	-2.03	38763.69	68.65
	50 50 -	413.500	-3.21	61267.32	-2.04	38695.11	68.75
	50 55 -	412.491	-3.21	61268.06	-2.02	38626.46	
	51 0 0	411.477	-3.22	61268.80	-2.01	38557.71	

Degrees of the Earth.

Places.	Latitudes.	Sum of the 3 equat. 4ths of DG being subtrac- tive.	Supple- mentary correction for deg. of great circles.	Degrees of great circles perpendicular to the meridian cor- rected.	Supple- mentary correction for deg. of long.	Degrees of longitude corrected.
				Diff.		Diff.
Greenwich	51° 0' 0"	411.477	-3.22	61268.80	0.74	38557.71
	51° 5' -	410.459	-3.22	61269.54	0.74	38488.88
	51° 10' -	409.437	-3.25	61270.28	0.74	38419.97
	51° 15' -	408.408	-3.29	61271.02	0.74	38350.97
	51° 20' -	407.387	-3.29	61271.76	0.74	38281.90
	51° 25' -	406.357	-3.30	61272.50	0.74	38212.73
	51° 28' 40"	405.601	-3.30	61273.04	0.54	38161.96
	51° 30' -	405.320	-3.31	61273.24	0.20	38143.48
	51° 35' -	404.291	-3.32	61273.98	0.74	38074.17
	51° 40' -	403.253	-3.32	61274.72	0.74	38004.76
	51° 45' -	402.213	-3.37	61275.46	0.74	37935.28
	51° 50' -	401.168	-3.37	61276.21	0.75	37865.69
	51° 55' -	400.128	-3.38	61276.95	0.74	37796.01
	52° 0' 0"	399.080	-3.40	61277.69	0.74	37726.31
				8.94	-2.09	37656.61
	53° - -	386.325	-3.35	61286.63	8.95	36883.22
	54° - -	373.232	-3.73	61295.58	8.97	36028.63
	55° - -	359.851	-3.90	61304.55	8.97	35162.84
	56° - -	346.187	-4.07	61313.52	8.97	34286.09
	57° - -	332.301	-4.23	61322.46	8.94	33398.61
	58° - -	318.213	-4.39	61331.37	8.91	32500.67
	58° 42' 47.4"	308.075	-4.51	61337.70	6.33	31854.07
	59° - -	303.963	-4.55	61340.23	2.53	31592.55
	60° - -	289.569	-4.69	61349.03	8.80	30674.52
Lapland				42.39	-2.34	30674.52
	65° - -	217.320	-5.28	61391.42	10.69	25945.14
	66° 20' -	198.361	-5.39	61402.11	27.58	24547.73
	70° - -	148.229	-5.47	61429.69	32.38	21010.19
	75° - -	87.821	-4.89	61462.07	25.39	15907.55
	80° - -	41.569	-3.81	61487.46	13.98	10677.11
Pole	85° - -	10.309	-2.16	61501.44	5.16	5360.20
	90° 0' 0"	0.000	±0.00	61506.60	±0.00	0000.00

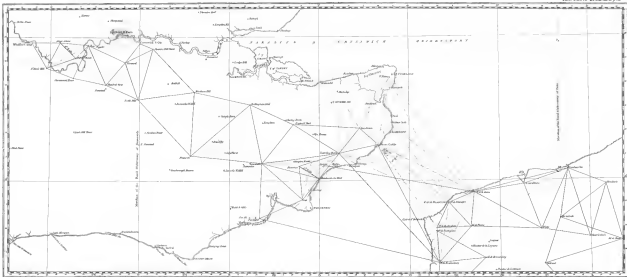
N. B. The degree of the meridian, whose center answers to the latitude of $43^{\circ} 32'$, is in length 60714.54 fathoms.

Table of the degrees of the Earth contained in the Apocrypha of Mr. Newton, where the measure of the degrees of the meridian above them is repeated below the ratio of the 4th power, or square of the ratio of the line of the sine.

Latitude	Length of degree of the meridian in degrees	Ratio of the 4th power of the sine of the latitude	Degrees of great circles contained in the meridian, and their ratio of the 4th power of the sine of the latitude	Degrees of great circles contained in the meridian, and their ratio of the 4th power of the sine of the latitude										Degrees of the meridian			Latitude	
				Lat.	1st power	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	4	2	2	2	4	8	16	32	64	128	256	512	1024	2048	4096	2	2
3	3	9	3	3	3	9	27	81	243	729	2187	6723	20301	62916	197727	622080	3	3
4	4	16	4	4	4	16	64	256	1024	4096	16384	65536	262144	1032192	4096000	16384000	4	4
5	5	25	5	5	5	25	125	625	3125	15625	78125	390625	1953125	9765625	48828125	244140625	5	5
6	6	36	6	6	6	36	216	1296	7776	46656	279936	1679616	10137600	61664640	380163840	2343603200	6	6
7	7	49	7	7	7	49	343	2401	16807	117649	823543	5724253	39810721	278692049	1950137949	13685790449	7	7
8	8	64	8	8	8	64	512	4096	32768	262144	2097152	16777216	134218752	1073743360	8589890560	68719436800	8	8
9	9	81	9	9	9	81	729	59049	531441	4782969	43046721	387420497	3486788473	31381096257	282429566313	2541855091329	9	9
10	10	100	10	10	10	100	1000	10000	100000	1000000	10000000	100000000	1000000000	10000000000	100000000000	1000000000000	10	10
11	11	121	11	11	11	121	1331	17716	241375	3177143	41334861	537353193	7005591511	91072649641	1182944445331	15378277789301	11	11
12	12	144	12	12	12	144	1728	29859	491328	8153727	134217728	2239488384	37324869376	618800639936	10101130237440	161698083801600	12	12
13	13	169	13	13	13	169	2197	40122	706993	12914657	235315153	4400377591	83307186227	1574020039559	30036330752281	57053328230449	13	13
14	14	196	14	14	14	196	2744	59136	1054136	18817856	340121344	6362104320	120500000000	2288800000000	43988800000000	843808000000000	14	14
15	15	225	15	15	15	225	3375	80625	1771463	40000000	900000000	20250000000	455625000000	10251562500000	231459375000000	5200134375000000	15	15
16	16	256	16	16	16	256	4096	100000	2621440	67108864	1730304000	44773893120	1159751936000	29748298752000	758617567360000	19461249792000000	16	16
17	17	289	17	17	17	289	4913	128129	3281797	84459663	218534137	5686387553	14680407631	37973059841	97624945581	253824867451	17	17
18	18	324	18	18	18	324	5832	157464	4049536	104857600	2716729600	69982976000	1811617280000	46791648000000	1204781184000000	31119150080000000	18	18
19	19	361	19	19	19	361	6859	186659	4747633	12167189	31116927	795425193	2018505497	5147114193	13182096081	33957439701	19	19
20	20	400	20	20	20	400	8000	240000	6000000	150000000	3750000000	93750000000	2343750000000	58593750000000	1464843750000000	36621093750000000	20	20
21	21	441	21	21	21	441	9261	282429	7125833	18014567	45838929	117591663	298229261	755395573	1923428433	4897913421	21	21
22	22	484	22	22	22	484	10648	329856	8421856	21494016	54730048	139325120	3518128000	89250304000	2276257600000	58162444800000	22	22
23	23	529	23	23	23	529	12167	381329	9717433	24858167	62145413	155363529	393408821	983522053	2458805133	6147012821	23	23
24	24	576	24	24	24	576	13824	442368	11139328	28349056	71372640	178431600	4460790400	111519760000	2837994400000	71544860000000	24	24
25	25	625	25	25	25	625	15625	516875	12917187	40000000	900000000	20250000000	455625000000	10251562500000	231459375000000	5200134375000000	25	25
26	26	676	26	26	26	676	17716	591360	14828864	37513600	94784000	2369600000	59239680000	1480991360000	37513600000000	947840000000000	26	26
27	27	729	27	27	27	729	19683	678129	17191063	43981687	112457221	286143053	720361781	1851916481	4654831201	11637078001	27	27
28	28	784	28	28	28	784	21952	778240	19950080	50870272	129151680	3253843200	82598976000	2106572480000	52760873600000	1339582400000000	28	28
29	29	841	29	29	29	841	24649	888129	22701263	57953207	145883021	369707553	924268881	2310672161	5776680401	14441700961	29	29
30	30	900	30	30	30	900	27000	1000000	25000000	625000000	15625000000	390625000000	9765625000000	244140625000000	6103515625000000	152587890625000000	30	30
31	31	961	31	31	31	961	29791	1126729	28743993	72860187	184190473	460480189	1151204473	2877911681	7195019201	18007548481	31	31
32	32	1024	32	32	32	1024	32768	1280000	32768000	838860800	21309977600	539604070400	13690240000000	346206720000000	8709363200000000	218620288000000000	32	32
33	33	1089	33	33	33	1089	35937	1404969	36058233	91351857	231389781	580967353	1452418381	3631086961	9077717401	22694593601	33	33
34	34	1156	34	34	34	1156	39304	1543696	39673024	101149696	252979200	6324480000	158112000000	3952800000000	98820000000000	24701000000000000	34	34
35	35	1225	35	35	35	1225	42875	1816563	46664063	116660157	291650393	729125981	1822814953	4557037381	11392593453	28481483601	35	35
36	36	1296	36	36	36	1296	46656	2000000	51994368	130079360	325273600	8127001600	203175040000	50793600000000	1269840000000000	31746240000000000	36	36
37	37	1369	37	37	37	1369	50653	2202009	56353827	140915021	354257553	885664021	2214165053	5535412621	13838531581	34586326401	37	37
38	38	1444	38	38	38	1444	54880	2426240	62180096	157664256	394194688	9854867200	246371648000	6159341120000	153983528000000	38495360000000000	38	38
39	39	1521	39	39	39	1521	59319	2684379	68041373	172603427	431508521	1078771303	2696928261	6742320653	16855801621	42144504001	39	39
40	40	1600	40	40	40	1600	64000	2969600	75800000	189500000	4737500000	118437500000	29609375000000	7402343750000000	185058593750000000	462646400000000000	40	40
41	41	1681	41	41	41	1681	68821	3266129	83911263	212019287	530048721	1325121753	3312806453	8282096121	20707239801	51768102401	41	41
42	42	1764	42	42	42	1764	73824	3592384	92460160	234100480	585251200	14631280000	365782080000	91445520000000	22851380000000000	571634400000000000	42	42
43	43	1849	43	43	43	1849	79009	3950089	101652233	258140587	645351453	1613378621	4033446553	10083616381	25208040961	63020097601	43	43
44	44	1936	44	44	44	1936	84416	4340096	111961600	289004800	722512000	180628000000	4515700000000	112892500000000	28500000000000000	712500000000000000	44	44
45	45	2025	45	45	45	2025	90000	4762500	123437500	312500000	7812500000	195312500000	48828125000000	12207031250000000	305175781250000000	7629419062500000000	45	45
46	46	2116	46	46	46	2116	95840	5216640	136513280	341282560	8532070400	213301760000	54125000000000	13531250000000000	338281250000000000	845703125000000000	46	46
47	47	2209	47	47	47	2209	101953	569529	15040863	37602167	94005417	235013543	587533853	1468834621	3671586581	9179016401	47	47
48	48	2304	48	48	48	2304	108384	6201600	166272000	421107200	1052768000	263192000000	65798000000000	16449500000000000	411237500000000000	10280900000000000000	48	48
49	49	2401	49	49	49	2401	115137	6740129	183681233	464203287	116682921	290127053	725317621	1813294053	4533235121	11333087601	49	49
50	50	2500	50	50	50	2500	122200	7316000	202080000	505200000	12630000000	315750000000	78937500000000	19734375000000000	493359375000000000	12333984375000000000	50	50
51	51	2601	51	51	51	2601	129579	7932329	221852633	554731687	138682921	346707281	866768121	2166920653	5417301621	13543253601	51	51
52	52	2704	52	52	52	2704	137296	8590080	243200000	610240000	15256000000	381400000000	95350000000000	23837500000000000	595343750000000000	14884843750000000000	52	52
53	53	2809	53	53	53	2809	145353	9282249	266251233	668128087	166782020							

MAP showing nearly the DISPOSITION of the TRIANGLES, where only it is proposed to determine the Relative Situations of the Royal Observatories of GREENWICH and PARIS.

Scale from 1 to 1,000,000 feet



Scale from 1 to 1,000,000 feet

Scale from 1 to 1,000,000 feet

Scale from 1 to 1,000,000 feet

Reynoldsman: Told the Judge of the Book

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